CSE 321 Discrete Structures

January 29, 2010

Lecture 10: Program Correctness

Announcements

- Homework #4 will be posted tomorrow
  - Slightly shorter, because of the midterm
- Midterm: Friday in class (1:30-2:30)
- Makeup midterm: Wednesday, Feb. 3, CSE 503, 3:30pm-4:30pm
  - If you plan to take the midterm on Wednesday, please send me email in advance
  - (Backup: 4:30-5:30; hopefully we won't need)

What to Study for the Midterm

- Read ALL lecture notes
- Read ALL handouts
- Review the homework solutions
- Rosen: read chapters 1, 4, 7.1, and 7.2

Binary Search

```
/* assume a[0] ≤ a[1] ≤ … ≤ a[n-1] */
/* find i  in the array a[ ]: */
/* either find i such that a[i] = x */
/* or find i such that a[i] < x < a[i+1] */
```

Simplified Binary Search

We will assume first that a[0] ≤ x

```
int i = 0; int j = n;
        while (i+1 < j) {
            int k = (i + j) / 2;
            if (x < a[k]) j = k;
            if (x >= a[k]) i = k;
        }
```

Simplified Binary Search

Precondition:

$$a[0] \leq x \ \wedge \ a[n] = \infty \ \wedge$$
$$\forall u, v \in \{0..n\text{-}1\}.\ u < v \rightarrow a[u] \leq a[v]$$

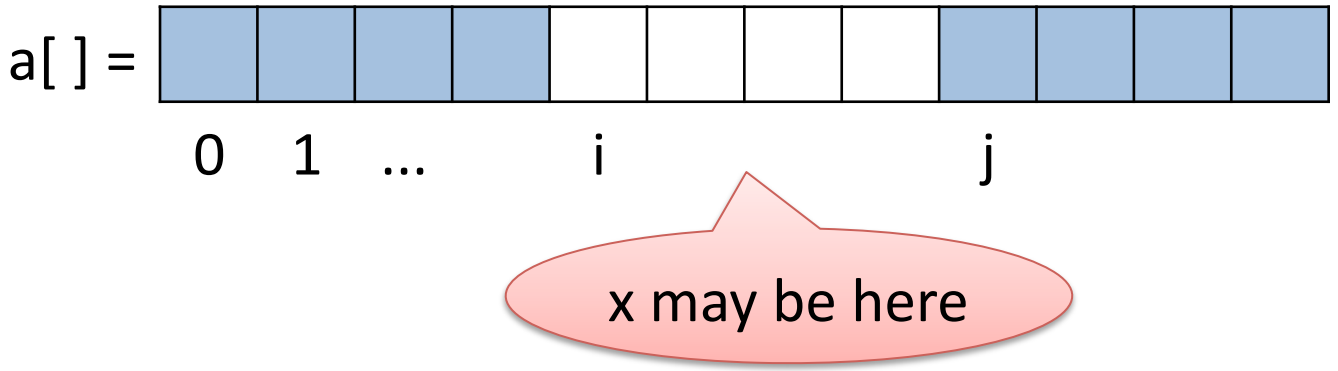
Postcondition:

$$i+1=j \ \wedge \ a[i] \leq x < a[j]$$

Proof (Assuming a[0] ≤ x)

Loop invariant:

$$(\text{precondition}) \wedge i < j \wedge a[i] \leq x < a[j]$$



(Prove partial correctness on the white board)

Binary Search

Now we drop the assumption that a[0] ≤ x

```
int i = -1; int j = n;
        while (i+1 < j) {
            int k = (i + j) / 2;
            if (x <= a[k]) j = k;
            if (x >= a[k]) i = k;
        }
```

Binary Search

Precondition:

$$a[-1] = -\infty \ \wedge\ a[n] = \infty \ \wedge$$
$$\forall u, v \in \{0..n-1\}.\ u < v \rightarrow a[u] \leq a[v]$$

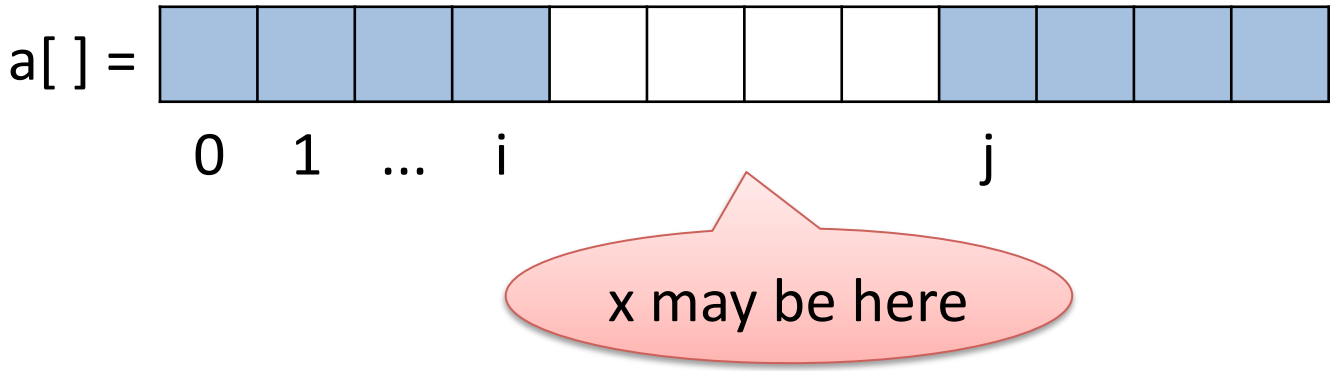
Postcondition:

$$(i+1=j \ \wedge\ a[i] < x < a[j]) \ \vee\ (i=j \ \wedge\ a[i]=x)$$

Proof

Loop invariant:

$$(\text{precondition}) \wedge (i<j \wedge a[i] < x < a[j]) \vee (i=j \wedge a[i]=x)$$



(Prove partial correctness on the white board)