CSE 321 Discrete Structures

January 15, 2010

Lecture 06: Practical Proofs

Announcements

- Reading from the textbook
  - Material covered so far: Chapter 1 (read !)
  - Material you need to know: Chapter 2 (read !)
  - Material for the next 3 lectures: Chapter 4 (read !)
- Homework 2
  - New due date: Friday, Jan 22
- Martin Luther King Jr. Day, Mon., Jan 18

$\forall x\, (\text{UniversityHoliday}(x) \rightarrow \text{NoClass}(x)) \wedge \text{UniversityHoliday}(\text{Monday}) \rightarrow \text{NoClass}(\text{Monday})$

Outline

- Proof methods today (simple: read Ch. 1)
  - Direct proof
  - Contrapositive proof
  - Proof by contradiction
  - Proof by equivalence
- Sets and functions today (simple: read Ch. 2)
- Proof methods next lectures (subtle: Ch. 4):
  - Induction
  - Complete induction
  - Structural induction

Direct Proof

- If n is odd, then n^2 is odd

Definition
n is even if $n = 2k$ for some integer k
n is odd if $n = 2k+1$ for some integer k

Contrapositive

- Sometimes it is easier to prove $\neg q \rightarrow \neg p$ than it is to prove $p \rightarrow q$
- Prove that if $ab \leq n$ then $a \leq n^{1/2}$ or $b \leq n^{1/2}$

Proof by contradiction

- Suppose we want to prove *p* is true.
- Assume *p* is false, and derive a contradiction

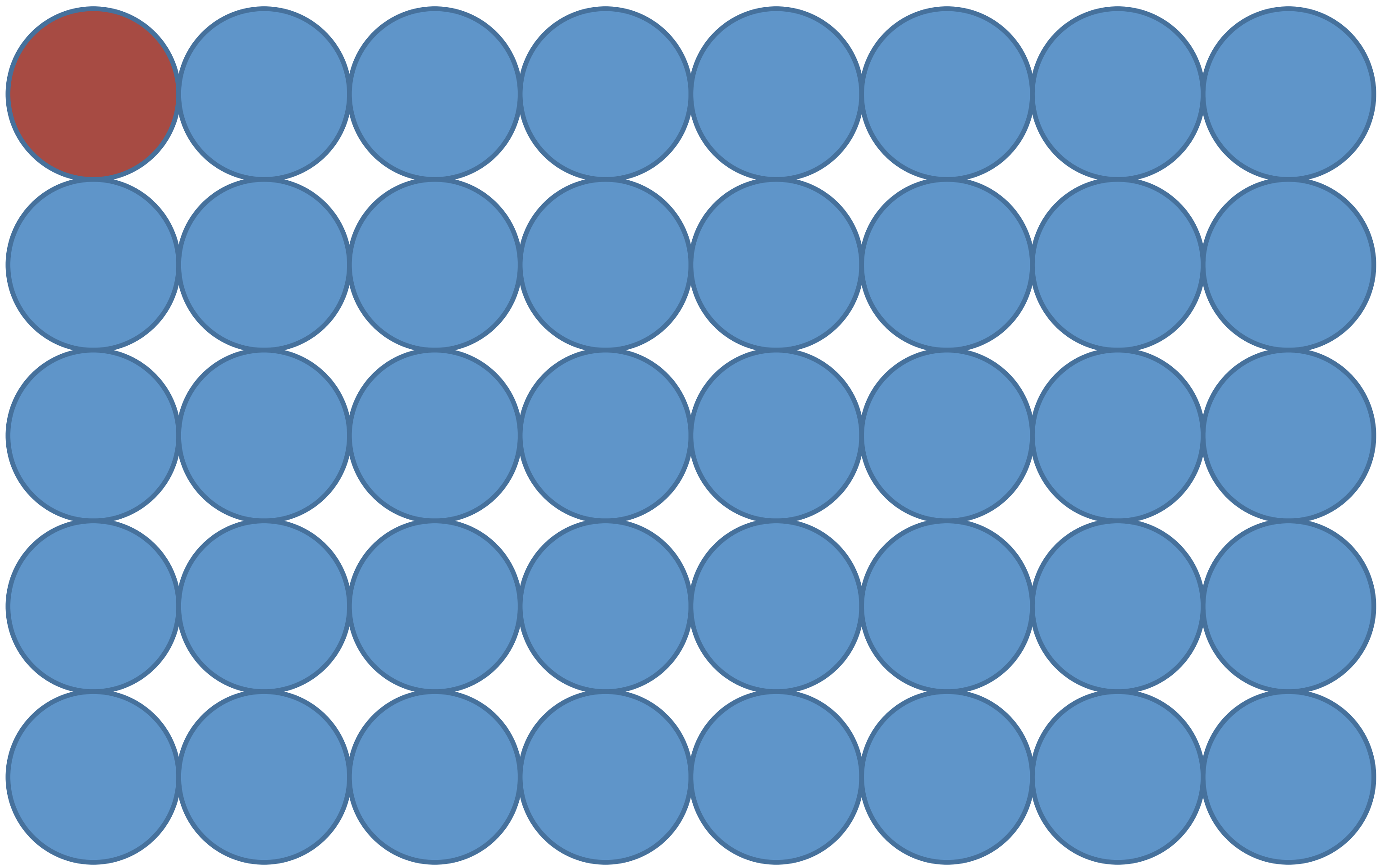
Contradiction example

- Show that at least four of any 22 days must fall on the same day of the week

Equivalence Proof

- To show $p_1 \leftrightarrow p_2 \leftrightarrow p_3$, we show $p_1 \rightarrow p_2$, $p_2 \rightarrow p_3$, and $p_3 \rightarrow p_1$
- Show that the following are equivalent
  - p_1: n is even
  - p_2: n-1 is odd
  - p_3: n^2 is even

The Game of Chomp

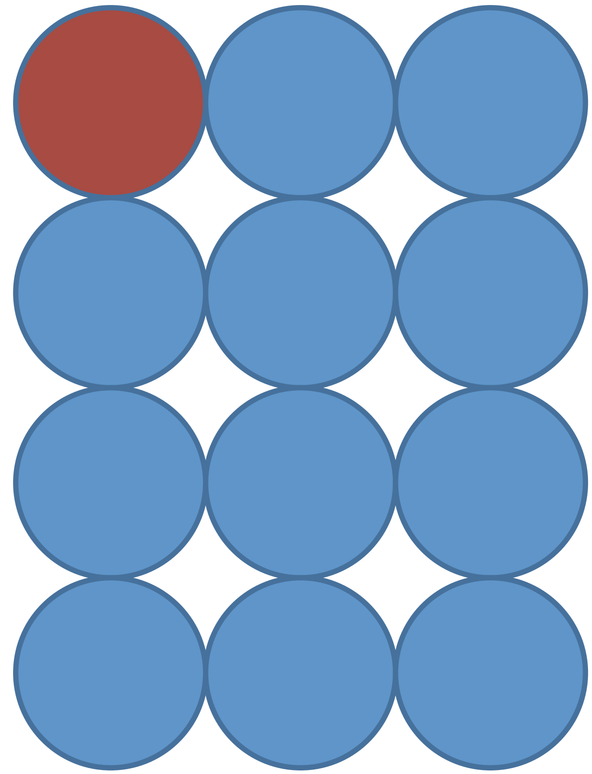


Theorem: The first player can always win in an $n \times m$ game

- Every position is a forced win for player A or player B (this fact will be used without proof)
- Any finite length, deterministic game with no ties is a win for player A or player B under optimal play

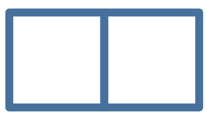
Proof

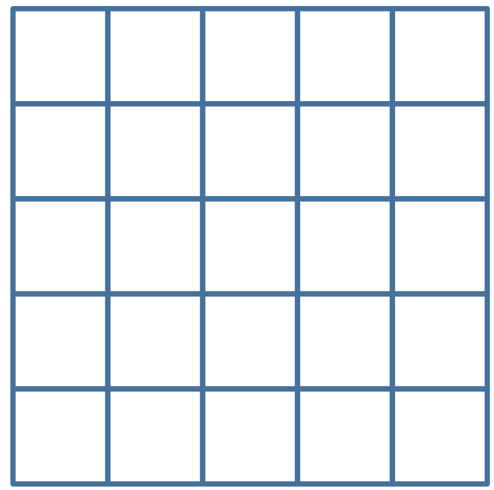
- Consider taking the lower right cell
  - If this is a forced win for A, then done
  - Otherwise, B has a move m that is a forced win for B, so if A started with this move, A would have a forced win

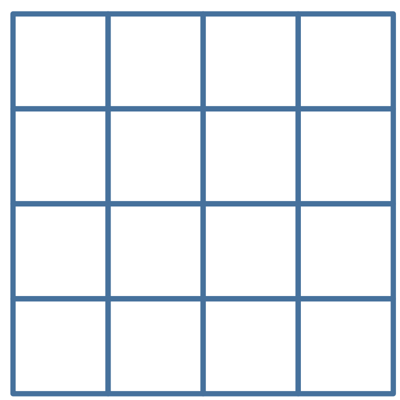


Tiling problems

- Can an $n \times n$ checkerboard be tiled with 2×1 tiles?

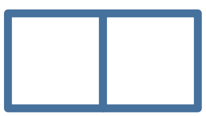


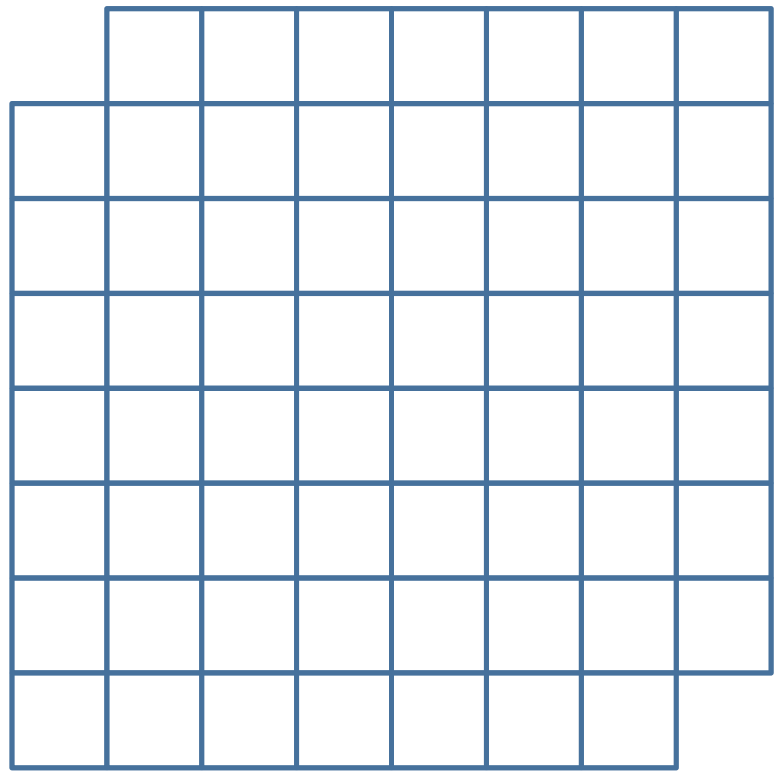




8× 8 Checkerboard with two corners removed

- Can an 8 × 8 checkerboard with upper left and lower right corners removed be tiled with 2 × 1 tiles?





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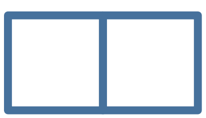
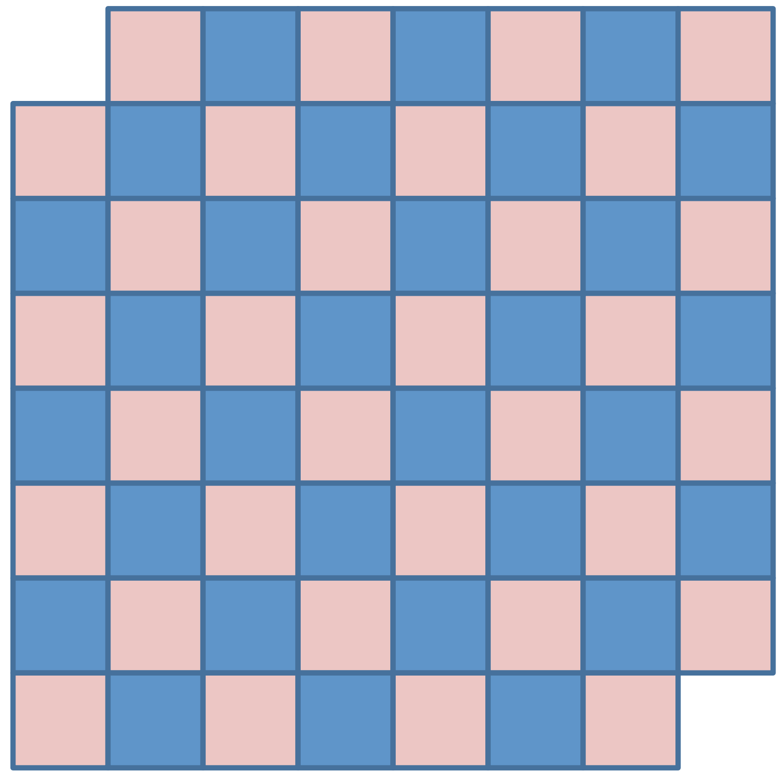
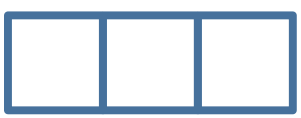
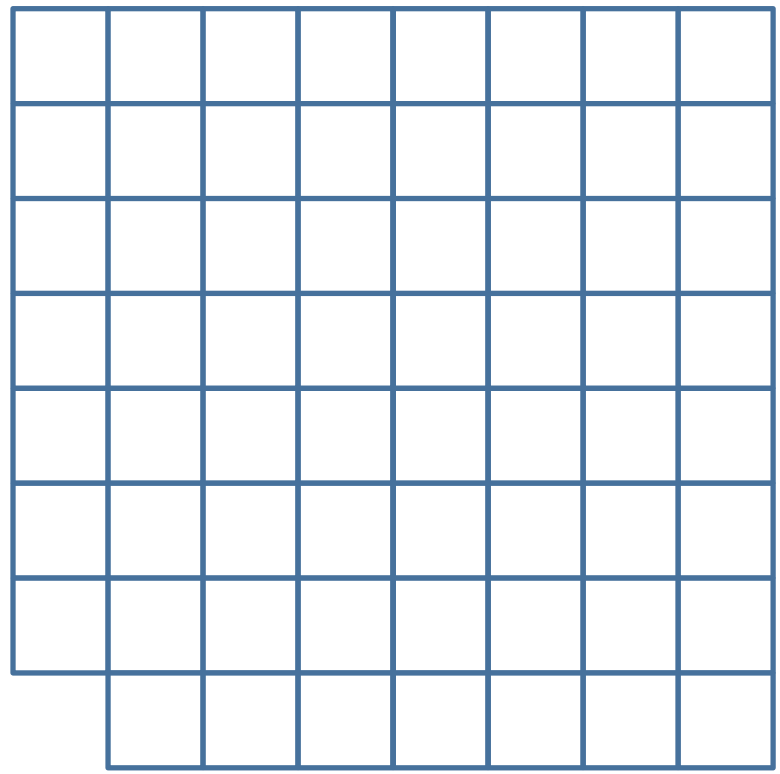




8× 8 Checkerboard with one corner removed

- Can an 8 × 8 checkerboard with one corner removed be tiled with 3 × 1 tiles?





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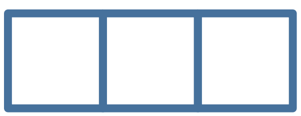
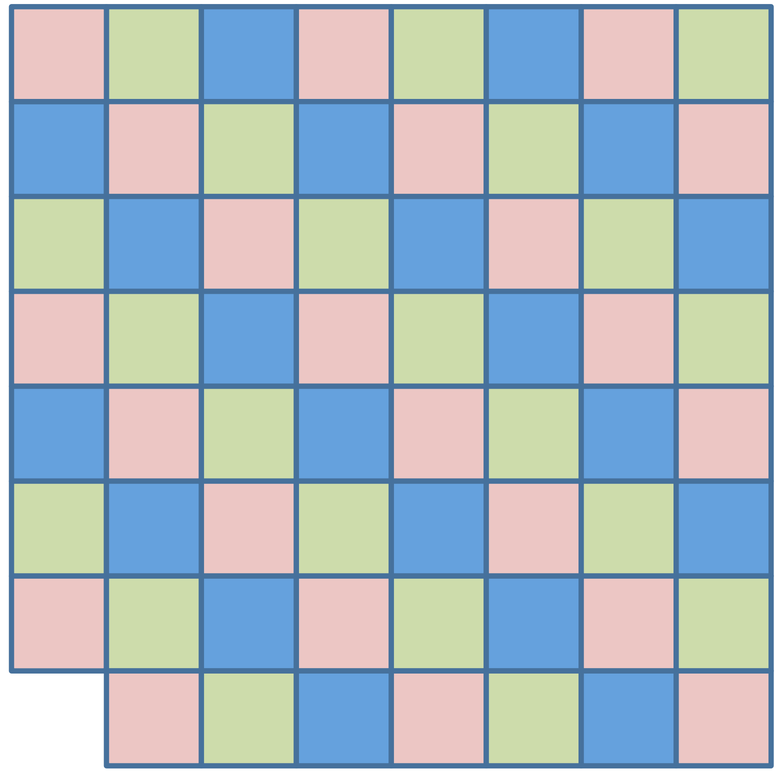
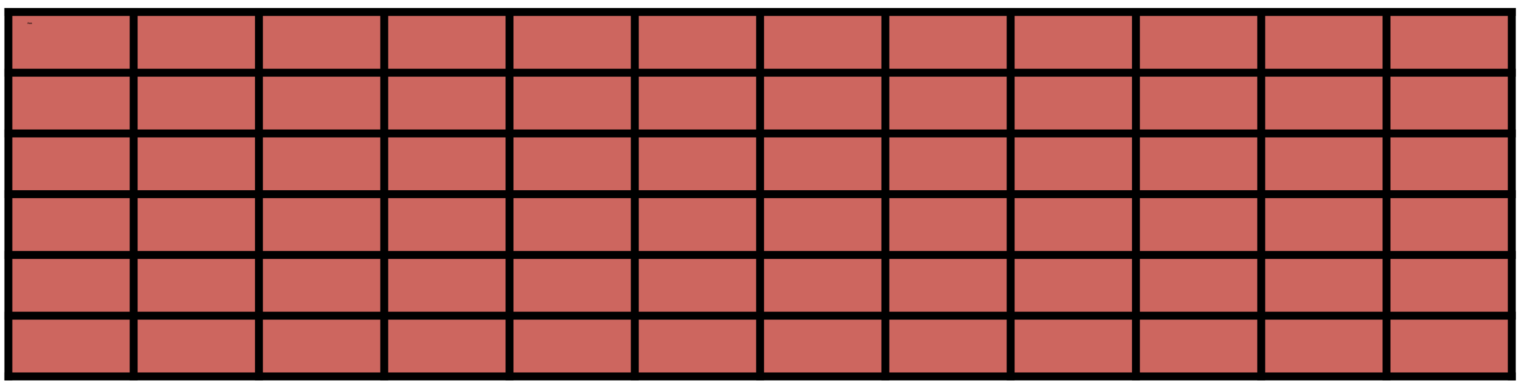




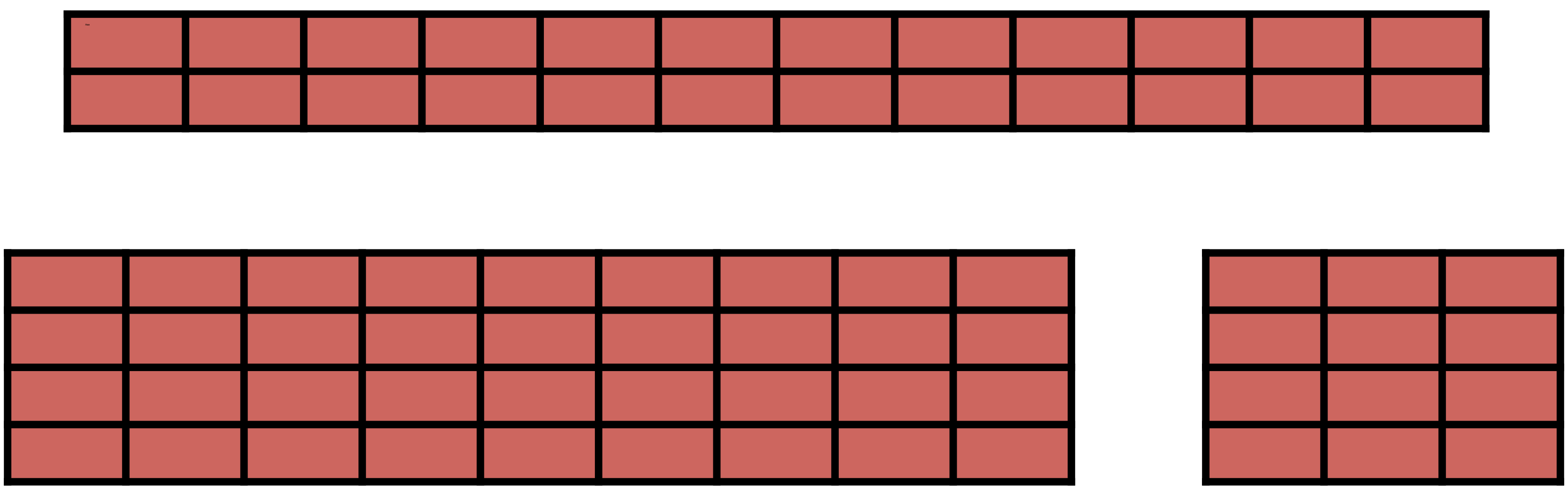
The Chocolate Bar Problem

- You have a 6 × 12 chocolate bar
- You want to split it into 72 pieces
- What is the minimum number of splits ?



The Chocolate Bar Problem

- After two splits:



Set Theory

- Formal treatment dates from late 19th century
- Direct ties between set theory and logic
- Important foundational language

Set Theory

Georg Cantor 1845-1918

Definition: A set is an unordered collection of objects

Definitions

- A and B are *equal* if they have the same elements

$$A = B \equiv \forall x\,(x \in A \leftrightarrow x \in B)$$

- A is a *subset* of B if every element of A is also in B

$$A \subseteq B \equiv \forall x\,(x \in A \rightarrow x \in B)$$

Empty Set and Power Set

Cartesian Product : $A \times B$

$$A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$$

Set operations

$$A \cup B = \{ x \mid x \in A \vee x \in B \}$$

$$A \cap B = \{ x \mid x \in A \wedge x \in B \}$$

$$A - B = \{ x \mid x \in A \wedge x \notin B \}$$

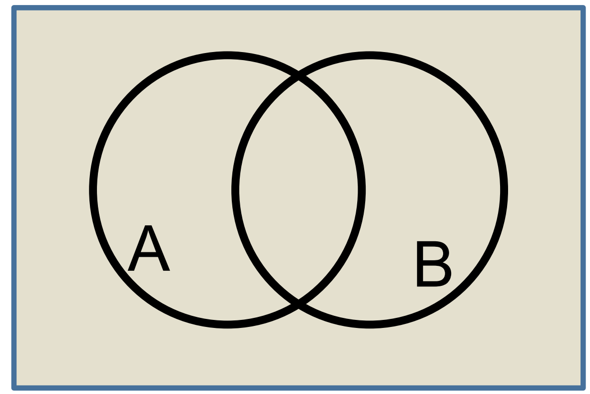
$$A \oplus B = \{ x \mid x \in A \oplus x \in B \}$$

$$\overline{A} = \{ x \mid x \notin A \}$$

De Morgan’s Laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

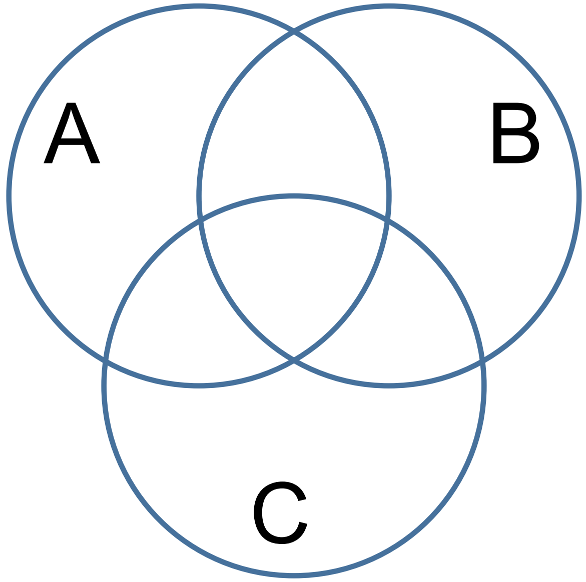
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

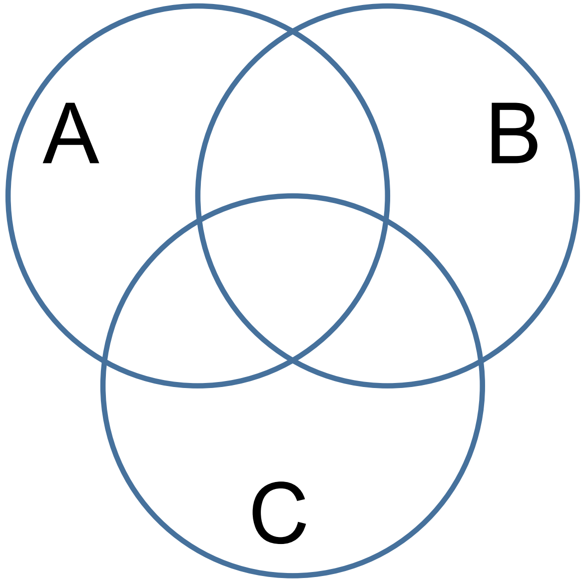


Proof technique:
To show C = D show
$x \in C \rightarrow x \in D$ and
$x \in D \rightarrow x \in C$

Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$





Russell's Paradox

$$S = \{ x \mid x \notin x \}$$

How do we "solve" the paradox ?