CSE 321 Discrete Structures

January 4, 2010

Lecture 01

Propositional Logic

About the course

- From the CSE catalog:
  - **CSE 321 Discrete Structures (4)** Fundamentals of set theory, graph theory, enumeration, and algebraic structures, with applications in computing. Prerequisite: CSE 143; either MATH 126, MATH 129, or MATH 136.
- What I think the course is about:
  - Foundational structures for the practice of computer science and engineering

Why this material is important

- Language and formalism for expressing ideas in computing
- Fundamental tasks in computing
  - Translating imprecise specification into a working system
  - Getting the details right

Topic List

- Logic/boolean algebra: hardware design, testing, artificial intelligence, databases, software engineering
- Mathematical reasoning/induction: algorithm design, programming languages
- Number theory/probability: cryptography, security, algorithm design, machine learning
- Relations/relational algebra: databases
- Graph theory: networking, social networks, optimization

Administration

- Instructor
  - Dan Suciu
- Teaching Assistant
  - Andrew Hunter
- Quiz section: Thursdays
  - 1:30 – 2:20 MGH 242, or
  - 2:30 – 3:20 EEB 054
- Text: Rosen, Discrete Mathematics
  - 6th Edition preferred
  - 5th Edition okay
- Homework
  - Due Wednesdays (starting Jan 13)
- Exams
  - Midterms, Feb 5
  - Final, March 15, 2:30-4:20
- All course information posted on the web
- Sign up for the course mailing list

Grading

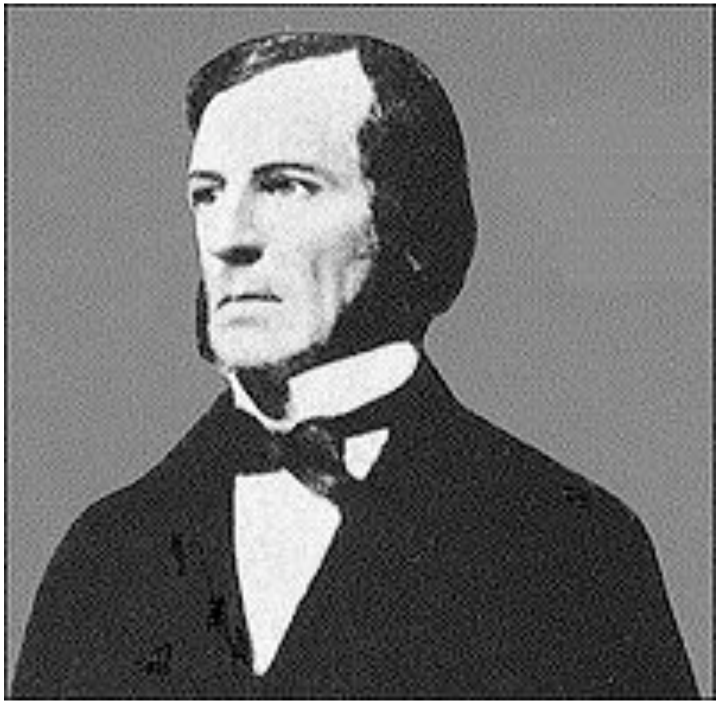
- 50% homeworks
- 20% midterm
- 30% final

Propositional Logic

- Talks about propositions
  - can be true or false
- Combine them, to obtain more complex propositions
  - Prove that these are true or false
- Not yet enough to describe foundations of mathematic and CS
  - Need *predicate logic* (future lecture)

Propositional Logic

George Boole (1815-1864)



Propositions

- A statement that has a truth value
- Which of the following are propositions?
  – The Washington State flag is red
  – It snowed in Whistler, BC on January 4, 2010.
  – Turn your homework in on Wednesday !
  – Why are we taking this class?
  – If n is an integer greater than two, then the equation $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c.
  – Every even integer greater than two can be written as the sum of two primes
  – This statement is false

– Propositional variables: *p, q, r, s, . . .*

– Truth values: **T** for true, **F** for false

Compound Propositions

- Negation (not) $\neg p$
- Conjunction (and) $p \wedge q$
- Disjunction (or) $p \vee q$
- Exclusive or $p \oplus q$
- Implication $p \rightarrow q$
- Biconditional $p \leftrightarrow q$

Truth Tables

p	$\neg p$
F	
T	

p	q	$p \wedge q$
F	F	
F	T	
T	F	
T	T	

p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

p	q	$p \oplus q$
F	F	
F	T	
T	F	
T	T	

x-or example: “you may have soup or salad with your entre”

Truth Tables

p	$\neg p$
F	T
T	F

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

Understanding complex propositions

- Either Harry finds the locket and Ron breaks his wand or Fred will not open a joke shop

Atomic propositions
h: Harry finds the locket
r: Ron breaks his wand
f: Fred opens a joke shop

$$(h \wedge r) \oplus \neg f$$

Understanding complex propositions with a truth table

h	r	f	h ∧ r	¬ f	(h ∧ r) ⊕ ¬ f
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Understanding complex propositions with a truth table

h	r	f	$h \wedge r$	$\neg f$	$(h \wedge r) \oplus \neg f$
F	F	F	F	T	T
F	F	T	F	F	F
F	T	F	F	T	T
F	T	T	F	F	F
T	F	F	F	T	T
T	F	T	F	F	F
T	T	F	T	T	F
T	T	T	T	F	T

Aside: Number of binary operators

- How many different binary operators are there on atomic propositions?

p	q	p op q
F	F	?
F	T	?
T	F	?
T	T	?

Answer: $2^4 = 16$

$p \rightarrow q$

- Implication
  - *p* implies *q*
  - whenever *p* is true *q* must be true
  - if *p* then *q*
  - *q* if *p*
  - *p* is sufficient for *q*
  - *p* only if *q*

p	q	$p \rightarrow q$
F	F	
F	T	
T	F	
T	T	

$p \rightarrow q$

- Implication
  - p implies q
  - whenever p is true q must be true
  - if p then q
  - q if p
  - p is sufficient for q
  - p only if q

p	q	p → q
F	F	T
F	T	T
T	F	F
T	T	T

True or False ?

- If it rains then the pavement gets wet
- If turn in your homework late then you will get 25% extra credit
- If pigs can whistle then horses can fly

True or False ?

- If it rains then the pavement gets wet

  T

- If turn in your homework late then you will get 25% extra credit

  F

- If pigs can whistle then horses can fly

  T

Converse, Contrapositive, Inverse

- Implication: $p \rightarrow q$
- Converse: $q \rightarrow p$
- Contrapositive: $\neg q \rightarrow \neg p$
- Inverse: $\neg p \rightarrow \neg q$

- Are these the same?

Example
p: “x is divisible by 2”
q: “x is divisible by 4”

Biconditional $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	p ↔ q
F	F	T
F	T	F
T	F	F
T	T	T

English and Logic

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old
  - q: you can ride the roller coaster
  - r: you are under 4 feet tall
  - s: you are older than 16

$$(r \wedge \neg s) \rightarrow \neg q$$

Application: Boolean Searches

- Google for Michael Jordan
  - I mean, of course, the leading researcher in machine learning, currently professor at Berkeley
- Type: “Michael Jordan”
  - No luck: the web seems obsessed with basketball…
- Type: “Michael Jordan –basketball”
  - Now we get it (4^{th} answer)

Means “not”