CSE 321: Discrete Structures
Assignment #1
January 7, 2009
Due: Wednesday, January 14

Reading Assignment: Read Sections 1.1 - 1.4 carefully (make sure that you understand the examples).

Problems:

1. Let p, q, r be the propositions.

    p You get sued by the Motion Picture Association of America (MPAA).

    q You illegally download Dumb and Dumber 2 (DD2).

    r You go to prison for 2 years.

    Write the following statements using p, q, r, and logical connectives.

    (a) You go to prison for 2 years, but you do not illegally download DD2.

    (b) You are sued by the MPAA, you illegally download DD2, and you go to prison for 2 years.

    (c) To get sued by the MPAA, it is necessary for you to illegally download DD2.

    (d) You are sued by the MPAA, but you dont illegally download DD2; nevertheless, you go to prison for 2 years.

    (e) Getting sued by the MPAA and illegally downloading DD2 is sufficient to get you sent to prison for 2 years.

    (f) You will go to prison for 2 years if and only if you either illegally download DD2 or you are sued by the MPAA.

2. Write each of these statements in the form if p then q in English. [**Hint**: Refer to the list of common ways to express conditional statements provided in Section 1.1]

    (a) I will remember to send you the address only if you send me an e-mail message.

    (b) To be a citizen of this country, it is sufficient that you were born in the U.S.

    (c) If you keep your textbook, it will be a useful reference in your future courses.

    (d) That you get the job implies that you had the best résumé.

    (e) The beach erodes whenever there is a storm.

    (f) It is necessary to have a valid password to log on to the server.

    (g) You will reach the summit unless you begin your climb too late.

3. State in English the converse and contrapositive of each of the following implications:

   (a) If a is pushed onto the stack before b, then b is popped before a.

   (b) If the input is correct and the program terminates, then the output is correct. (Be sure to use De Morgan's Law to simplify the contrapositive.)

4. On the island of Homvurkia, knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described below. If you cannot determine what these two people are, can you draw any conclusions?

   (a) A says "The two of us are both knights" and B says "A is a knave."

   (b) Both A and B say "I am a knight."

5. The following two statements form the basis of the most important methods for automated theorem proving. Use truth tables to prove that they are tautologies.

   (a) Resolution: $((p \lor q) \land (\neg q \lor r)) \to (p \lor r)$

   (b) Modus ponens: $((p \land (p \to q)) \to q$

6. Show that Modus ponens is a tautology without using a truth table. Show each step and indicate which logical equivalences you use.

7. Without using a truth table, show that $(p \to q) \lor (p \to r)$ and $p \to (q \lor r)$ are logically equivalent.

8. Give the negation of each of the following statements.

   (a) All good students study hard.

   (b) Some birds fly south for the winter.

   (c) No liberal arts majors are unable to use a computer.

   (d) $\forall x \exists y \ \ x = y^2$

9. **Extra Credit:** You have two memory registers, each with the same number of bits. You have an operation, XOR(R1,R2), which takes two registers, R1 and R2, performs bitwise $\oplus$ between them, and stores the result in R1. Show how you can swap the contents of the two registers using a sequence of XORs without temporary memory registers. Explain why this works.