Discrete Structures

Functions

Chapter 2, Section 2.3

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Functions

- $f : A \to B$**:** A function from A to B is an assignment of exactly one element of B to each element of A.

  - A is the domain of f and B is the codomain of f.

  - If $f(a) = b$, we say that b is the image of a and a is a pre-image of b. The range of f i the set of all images of elements of A.

  - f maps from A to B.

- $f_1 + f_2, f_1 f_2$**:** Let f_1 and f_2 be functions from A to R. Then
  $(f_1 + f_2)(x) = f_1(x) + f_2(x)$,
  $(f_1 f_2)(x) = f_1(x) f_2(x)$

Functions

- **Injection:** Function f is said to be one-to-one, if and only if $f(x) = f(y)$ implies that $x = y$ for all x and y in the domain of f.

- Function f whose domain and codomain are subsets of the set of real numbers is called strictly increasing if $f(x) < f(y)$ whenever $x < y$ and x and y are in the domain of f (decreasing analogous).

- **Surjection:** Function f is said to be onto / surjective, if and only if for every element $b \epsilon B$ there is an element $a \epsilon A$ with $f(a) = b$.

- **Bijection:** Function f is a one-to-one correspondence, or bijection, if it is both one-to-one and onto.

- **Inverse function:** Let f be a one-to-one correspondence from A to B. The inverse function of f assigns to an element b in B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^-1. Hence, $f^-1(b) = a$ when $f(a) = b$.

Functions

- $f \circ g$**:** $g : A \to B$, $f : B \to C$. The composition of the functions f and g is defined by
  $(f \circ g)(a) = f(g(a))$

- $\lfloor x \rfloor$ The floor function assigns to the real number x the largest integer that is less than or equal to x.

- $\lceil x \rceil$ The ceiling function assigns to the real number x the smallest integer that is greater than or equal to x.