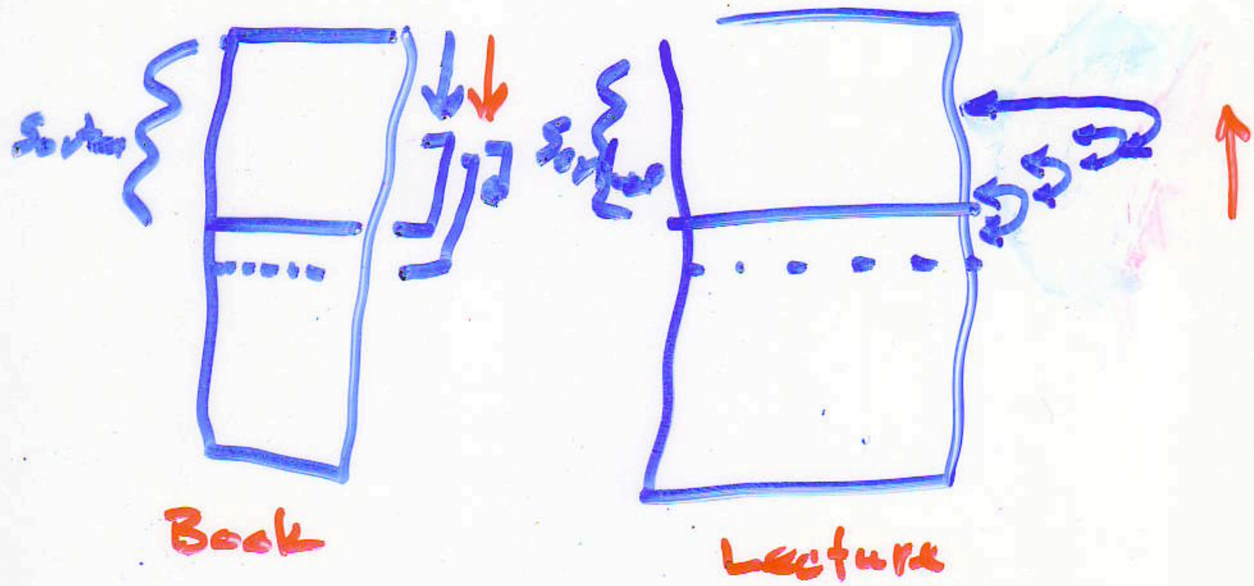
Insertion Sort




```
A: 0 .. n-1
For i = 1 .. n-1 {
  T = A[i]
  j = i-1
  While j >= 0 && T < A[j] {      "compare"
     A[j+1] = A[j]
  // A[j] = T                       "a swap"
     j = j-1
  }
  A[j+1] = T
}
```

or

Run Time

worst case $O(n^2)$ $\sim \frac{n^2}{2}$ swaps

compares = # swap + $n-1$

$n!$ "different" inputs

Assume all $n!$ inputs equally likely

Permutations

π 3 5 1 4 2

(i,j) an inversion in π
if $i<j$ but i after j in π

6 [
1,5
1,3
2,4
4,5
2,3
2,5

12345 : No inversions
54321 : $\binom{5}{2}$ inversions
(The max)

Swapping an adjacent pair of positions that are out of order decreases # of inversions by exactly 1.

# of swaps by insertion sort exactly # of inversions in its input

$$I_{i,j} = \begin{cases} 1 & \text{if } (i,j) \text{ is an inversion} \\ 0 & \text{if not} \end{cases}$$

# of Inversions $I = \sum_{i<j} I_{i,j}$

$$E(I) = \sum_{i<j} E(I_{i,j})$$

π .... i ... j ....

π' ..... j .... i ....

$$P(I_{i,j}=1) = E(I_{i,j}) = 1/2$$

(for every π where (i,j) is an inversion, there is π' where it's not.)

$$E(I) = \sum_{i<j} 1/2 = \binom{n}{2} \cdot 1/2$$

$\therefore$ Expected # of swaps $= \binom{n}{2}/2$

vs worst case $\binom{n}{2}$

i.e. average run time (assuming random input) is $\sim \frac{1}{2}$ of worst case

$E(X+Y) = E(X) + E(Y)$

$V(X+Y) \neq V(X) + V(Y)$ (in general)

but $=$ if X & Y are indp.

$E(X \cdot Y) \overset{?}{=} E(X) \cdot E(Y)$

$\neq$ in general, but

$=$ if indp.

both proved in text

Example

$X =$ 0/1 each w/ $p = 1/2$

$Y = X$

$E(X) = 1/2 = E(Y)$

$E(X \cdot Y) = \frac{1}{2} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 0 \cdot 0 =$

$= 1/2 \neq (1/2)^2 = E(X) \cdot E(Y)$