in n^2+1 $\exists$ monotone length $n+1$

(i_x, d_x)

i_x = length of longest increasing subsequence starting at position x

d_x = ditto for decreasing

Suppose Theorem false

then $1 \le i_x \le n$

$1 \le d_x \le n$

at most n^2 such pairs

Sequence length n^2+1

By P.H.P. $\exists$ $x < y$ st $(i_x, d_x) = (i_y, d_y)$

Suppose $m_x < m_y$

Then subsequence $m_x, m_y \ldots$

is increasing & length $i_y + 1 > i_x$

contradiction, so $m_x > m_y$

Then $m_x, m_y \ldots$ decreasing

seq of length $d_y + 1 \neq d_x$ 23-1

A **Permutation** is an ordered list of distinct elements

Permutations of $\{1, 2, 3\}$

1 2 3
1 3 2
2 1 3
2 3 1
3 1 2
3 2 1

$$\uparrow \ \uparrow \ \uparrow$$

$$3 \cdot 2 \cdot 1 = 6$$

Permutations of n things $= n!$

an r-Permutation of n things is an ordered list of r ~~dist~~ elements among n distinct choices

2-Permutations of 4 elements

1 2	2 1	3 1	4 1	
1 3	2 3	3 2	4 2	12
1 4	2 4	3 4	4 3	

$0 \le r \le n$

$$P(n,r) = n \cdot (n-1) \cdot \dots \overset{(n-r+1)}{(n-(r-1))}$$

$$= \prod_{i=0}^{r-1} (n-i)$$

$P(n,0) = 1$

$P(n,n) = n!$

$$P(n,r) = \frac{n!}{(n-r)!}$$

$0! = 1$

Combinations   unordered subsets

$C(n, r)$

# of different unordered subsets
of size r from a set of n
(distinct) objects

5 people

1 2	2 3	3 4	4 5
1 3	2 4	3 5	
1 4	2 5		
1 5			

$$C(n,2) = P(n,2)/2!$$

$$C(n,3) = P(n,3)/3!$$

$$= P(n,3)/P(3,3)$$

$$\binom{n}{r} = C(n,r) = P(n,r)/P(r,r)$$

$$= \frac{n!}{(n-r)!\, r!}$$

Eg $\binom{n}{2} = \frac{n(n-1)}{2 \cdot 1}$

$$= \frac{n\ (n-1)\ \cdots\ (n-(r-1))}{r\ \ (r-1)\ \ \cdots\ \ 1}$$

How many 10-bit strings have exactly 5 1's

$$C(10,5) = \frac{\cancel{10}^{2}\ 9\ \cancel{8}^{2}\ 7\ \cancel{6}}{\cancel{5}\ \cancel{4}\ \cancel{3}\ \cancel{2}\ 1}$$

$$= 63 \times 4 = 252$$

10 w 6 1's 10 w 4 0's

$$C(10,6) = C(10,4)$$

$$C(n,r) = C(n, n-r)$$

Binomial Theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$(x+y)(x+y)(x+y)\cdots(x+y)$$

$$\sum_{i=0}^{n} \binom{n}{i} = (1+1)^n = 2^n$$

$$1 \cdot x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2$$

23-5