Ch 5 Counting

How many WA license plates?

ddd LLL

$10 \times 10 \times 10 \times 26 \times 26 \times 26 = 17,576,000$

"Product Rule"

ordered series of choices

n_1 1st, for any 1st choice

n_2 ~~2~~ 2nd

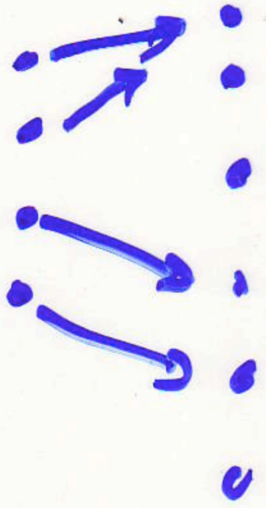
n_3 3rd

Then $n_1 \times n_2 \times n_3$ in total

License plates with no dup chars

$10 \cdot 9 \cdot 8 \quad 26 \cdot 25 \cdot 24$

How many functions $f : \mathbb{Z}_m \to \mathbb{Z}_n$



$n \cdot n \cdot n \cdots = n^m$

how many 1-1 functions?

if $n < m$ none

$n \geq m$

$$n\ (n-1)\cdot(n-2) \cdots (n-(m-1))$$

$$= \frac{n!}{(n-m)!}$$

Sum Rule

$A_1 \ldots A_n$ disjoint sets

# of choices is $\sum_{i=1}^{n} |A_i| = |\bigcup_{i=1}^{n} A_i|$

3 dolls

4 war toys 7 options

Inclusion-Exclusion

A, B

$|A \cup B| = |A| + |B| - |A \cap B|$

one wartoy is GI Joedoll

6 options

4+3-1

$\#$ of 8-bit strings that start with 1 <u>or</u> end with 00

$$\begin{array}{lrr}
1\,\_\,\_\,\_\,\_\,\_\,\_\,\_ & 2^7 & 128 \\
\_\,\_\,\_\,\_\,\_\,\_\,00 & \underline{+2^6} & \underline{+64} \\
 & & 192 \\
1\,\_\,\_\,\_\,\_\,\_\,00 & 2^5 & \underline{-32} \\
 & & 160
\end{array}$$

PHP

$K+1$ pigeons placed in
K pigeon holes means
at least one gets >1 pigeon

Eg: 2 or more in this room
born on same day
$\sim 50 > 31$

$\forall$ positive integer n $\exists\, q \in \mathbb{Z}$

st $n \cdot q = \underbrace{6666}_{\geq \text{three } 6\text{'s}} \cdots 00 \cdots 0$

$a_1 = 666$ mod n 0

$a_2 = 666666$ 1

$\vdots$ $\vdots$

$a_{n+1} = \underbrace{6 \cdots 6}_{3(n+1)}$ $n-1$

$$a_i \equiv a_j \pmod{n}$$

$i < j$

$$\frac{a_j - a_i}{n} = q$$

$$q \cdot n = \overbrace{6666\underbrace{0\ldots0}_{3i}}^{3j}$$

$x_1 \quad x_2 \quad \cdots \quad x_{n^2+1}$

distinct reals

$\exists$ monotonic subsequence

of length $\geq n+1$

$\underline{1} \quad 3 \quad \underline{2} \quad 9 \quad 5 \quad 0 \quad 6 \quad \underline{4} \quad \underline{8} \quad 7$

$n=3$

$n^2+1=10$