Suppose $\mathbb{R}^+$ are countable

.... $[0, 1)$ ...

0   $0.\underline{0}0000$

1   $0.2\underline{1}2121212121\ldots$

2   $0.12\underline{3}41234 56 \ldots$

3   $0.314\underline{1}5926535\,89793\ldots$

4   $0.2718\underline{9}99999999\ldots$

5   $0.27190\underline{0}005000\ldots$

⋮

$0.124281\ldots$

Let d_{ij} be j^{th} digit of i^{th} row

For digit d, let $\overline{d}$ be any other digit,

excluding $\{9, 0\}$

Let X be the real whose i^{th} digit is $\overline{d_{ii}}$.

Then X is <u>not</u> in the table since it differs from i^{th} row at i^{th} digit (at least)

[and avoids the nuisance $.4999\ldots = .5000\ldots$ issue]

$x = .4999\ldots$

$10x = 4.99\ldots$

$ 4.9\ldots$

$10x - x$

$9x = 4.5 \quad 000$

$x = \frac{4.5}{9} = \frac{9}{18} = \frac{1}{2} = .5$

Fact. if $A \subseteq B$ & B is countable then so is A.

The set of computable functions from $\mathbb{N} \to \{0,1\}$ is countable.

set of valid java $\subseteq \Sigma^*$

$B = \{ f \mid f : \mathbb{N} \to \{0,1\} \}$

suppose B is enumerable

	0	1	2	3		...
f_0	$\boxed{0}$	1	0	1	0	1
f_1	0	$\boxed{0}$	0	0	0	0
f_2	1	1	$\boxed{0}$	0	1	1 ...
f_3	1	0	1	0	$\boxed{1}$	0
⋮						$\boxed{?}$
d	1	1	1		0	? d

$d(i) = 1 - f_i(i)$

$d : \mathbb{N} \to \{0,1\}$ so $d \in B$

<u>but</u> d differs from f_i at i

$d(i) = 1 - f_i(i) \neq f_i(i)$

$d \neq f_i$

~~ex~~ finite alphabet $\Sigma = \{a, b, c\}$

Σ^* set of finite strings over Σ

λ	0
a	1
b	2
c	3
aa	4
ab	5
ac	6
ba	7
bb	8
bc	9
ca	10
cb	11
cc	12
aaa	13

Σ^* is countable

$$\forall x \in \Sigma^* \quad f(x) = \begin{cases} 0 & \text{if } x = \lambda \\ r \cdot f(y) + g(z) & \text{if } x = yz \text{ where } z \in \Sigma,\ y \in \Sigma^* \end{cases}$$

$f : \Sigma^* \to \mathbb{N}$

$r = |\Sigma|$

$g : \Sigma \to \{1, 2, \ldots r\}$ is a bijection

Exercise: Prove f is a bijection