Logical Equivalence

$$\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$$

$$\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$$

What if Domain is $\emptyset$

$$\forall x\, P(x) \equiv T$$

$$\exists x\, P(x) \equiv F$$

$$\exists x\, \neg P(x) = F$$

$\mathcal{U} \quad \mathbb{N}$

$E(x)$ even

$C(x)$ composite

$$\forall x\,(E(x) \to C(x))$$

$$\neg \forall x\,(E(x) \to C(x))$$

$$\equiv \exists x \neg (E(x) \to C(x))$$

$$\equiv \exists x \neg (\neg E(x) \vee C(x))$$

$$\equiv \exists x\,(E(x) \wedge \neg C(x))$$

"There is an even prime"

(2)

Logical Inference

if $\overbrace{\text{human}}^{p}$, then $\overbrace{\text{fallible}}^{q}$

$\overbrace{\text{I'm human}}$

$\therefore$ I'm fallible

$p \to q$

p

$\therefore\ q$

$(p \wedge (p \to q)) \to q$

Tautology

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syl Syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllo Syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution

h_1
h_2
h_3
$\therefore c$

$(h_1 \land h_2 \land h_3) \to c$

should be tautology

it's not sunny & colder than yesterday $\neg p \wedge q$

→ we swim only if sunny $r \to p$

if no swim then canoe $\neg r \to s$

if canoe then home by dark $s \to t$

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1. $\neg p \wedge q$ (hyp)
2. $\neg p$ simplification 1
3. $r \to p$ hyp
4. $\neg r$ modus tollens 2,3
5. $\neg r \to s$ hyp
6. s modus ponens 4,5
7. $s \to t$ hyp
8. t modus ponens 6,7

home by dark.

TABLE 2 Rules of Inference for Quantified Statements.	
Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization