predicate

X is even $\leftarrow E(x)$ propositional function

subject — predicate

is composite $\leftarrow C(x)$

is greater than $G(x,y)$ $x>y$

Universe or Domain of Discourse

Quantifiers

Existential $\exists$ "there exists"

Universal $\forall$ "for all"

$\exists x\, E(x)$ "there is some even nat"

$\forall x\, E(x)$ "all nats are even"

if universe is finite $x_1\, x_2 \ldots x_n$

$$\exists x\, E(x) \equiv E(x_1) \vee E(x_2) \vee \ldots \vee E(x_n)$$

$$\forall x\, E(x) \equiv E(x_1) \wedge E(x_2) \wedge \ldots \wedge E(x_n)$$

To Show "$\exists_x P(x)$"

find ~~one~~ example x_0 st $P(x_0)$

to show "$\forall x\ P(x)$"

to prove that $P(x)$ holds

for every x in domain

to show "$\neg \forall x\ P(x)$"

find x_0 in domain st $\neg P(x_0)$

counter example

E even $\qquad \mathbb{N}$

C composite $\qquad G(x,y): x > y$

$$\forall x \; E(x)$$

$$\forall x \; (E(x) \to C(x))$$

$$\forall x \big((G(x,2) \land E(x)) \to C(x) \big)$$

Digression

why should $\left\{ \begin{matrix} F \to T \\ F \to F \end{matrix} \right\}$ be true? (*)

A common case: a rule with exceptions

"all even numbers are composite, except 0, 2"

"all numbers are composite, except odd or ≤ 2"

Convenient:

$$\forall x \; \underbrace{\Big(\underbrace{\text{Unexceptional}(x)}_{\text{false for exceptions}} \to \text{Rule}(x) \Big)}_{\substack{\text{true for exceptions by (*)} \\ \text{(indp. of Rule}(x))}}$$

nested Quantifiers

$\forall x (\forall y\, P(x,y))$

$\forall x\, Q(x)$

when $Q(x) = \underline{\forall y\, P(x,y)}$

free variable

$\exists x\, \exists y\, P(x,y)$

bound variables

For all pairs x, y $P(x,y)$ is true

$\forall x \forall y\, P(x,y) \equiv \forall y \forall x\, P(x,y)$

$\exists x\, (\forall y\, P(x,y))$

$\not\equiv$

$\forall y\, (\exists x\, P(x,y))$

Domain $\mathbb{N}$

$\exists x\, \forall y\;\; y < x \leftarrow F$

$\forall y\, \exists x\;\; y < x \leftarrow T$ (say $x = y+1$

$\equiv$

two formulas are logically equivalent if they have the same truth value for <u>all</u> domains and <u>all</u> predicates

$\forall x \left[P(x) \wedge Q(x) \right]$

$\equiv \forall x \, P(x) \wedge \forall x \, Q(x)$

$\equiv \forall x \, P(x) \wedge \forall y \left(Q(y) \right)$

$\forall x \left(P(x) \vee Q(x) \right)$

$\not\equiv$

$\forall x \left(P(x) \right) \vee \forall x \left(Q(x) \right)$

$\mathbb{N}$

P = even

Q = odd