# CSE 321 Discrete Structures

Winter 2008 Lecture 22 Binary Relations

#### Announcements

- Readings
  - Relations
    - Section 8.1 Binary Relations
    - Section 8.2 n-Ary relations
    - Section 8.3 Representing Relations

Relations

### Highlights from Lecture 21

- Expectation of a Random Variable – Linearity of Expectation
- Average Case Analysis of Algorithms

# **Definition of Relations**

Let A and B be sets, A binary relation from A to B is a subset of  $A \times B$ 

Let A be a set, A binary relation on A is a subset of  $A \times A$  **Relation Examples** 

# **Properties of Relations**

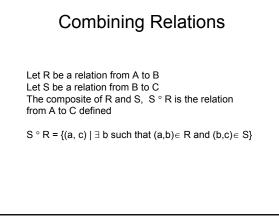
Let R be a relation on A

R is reflexive iff (a,a)  $\in$  R for every a  $\in$  A

R is symmetric iff  $(a,b) \in R$  implies  $(b, a) \in R$ 

R is antisymmetric iff  $(a,b) \in R$  and  $a \neq b$  implies  $(b,a) \notin R$ 

R is transitive iff  $(a,b) \in R$  and  $(b, c) \in R$  implies  $(a, c) \in R$ 



Examples (a,b)∈ Parent: b is a parent of a (a,b)∈ Sister: b is a sister of a What is Parent ° Sister? What is Sister ° Parent? S ° R = {(a, c) | ∃ b such that (a,b)∈ R and (b,c)∈ S}

# Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother express

Uncle: b is an uncle of a

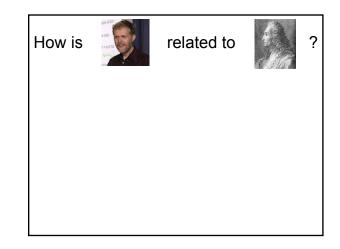
Cousin: b is a cousin of a

# Powers of a Relation

 $R^2 = R \, ^\circ R = \{(a, \, c) \mid \exists \ b \ \text{such that} \ (a, b) \in R \ \text{and} \ (b, c) \in R \}$ 

 $\mathsf{R}^{0} = \{(a,a) \mid a \in \mathsf{A}\}$ 

 $\begin{aligned} & \mathsf{R}^1 = \mathsf{R} \\ & \mathsf{R}^{\mathsf{n}+1} = \mathsf{R}^\mathsf{n} \, ^\circ \mathsf{R} \end{aligned}$ 

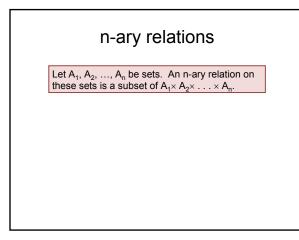


# From the Mathematics Geneology Project

Erhard Weigel Gottfried Leibniz Jacob Bernoulli Johann Bernoulli Johann Bernoulli Joseph Lagrange Jean-Baptiste Fourier Gustav Dirichlet Rudolf Lipschitz Felix Klein C. L. Ferdinand Lindemann Herman Minkowski Canstantin Caratheodory Georg Aumann Friedrich Bauer Manfred Paul Ermst Mayr Richard Anderson

### Transitivity and Composition

R is Transitive if and only if  $R^n \subseteq R$  for all  $n \geq 1$ 



# Relational databases

Knuth 328012098 CS 4.00   Von Neuman 481080220 CS 3.78   Von Neuman 481080220 Mathematics 3.78   Russell 238082388 Philosophy 3.85   Einstein 238001920 Physics 2.11   Newton 1727017 Mathematics 3.61   Karp 34882811 CS 3.98   Newton 1727017 Physics 3.61   Bernoulli 2921938 Mathematics 3.21	Student_Name	ID_Number	Major	GPA
Von Neuman 481080220 Mathematics 3.78   Russell 238082388 Philosophy 3.85   Einstein 238001920 Physics 2.11   Newton 1727017 Mathematics 3.61   Karp 34882811 CS 3.98   Newton 1727017 Physics 3.61   Bernoulli 2921938 Mathematics 3.21	Knuth	328012098	CS	4.00
Russell 238082388 Philosophy 3.85   Einstein 238001920 Physics 2.11   Newton 1727017 Mathematics 3.61   Karp 34882811 CS 3.98   Newton 1727017 Physics 3.61   Bernoulli 2921938 Mathematics 3.21	Von Neuman	481080220	CS	3.78
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Bernoulli 2921938 Mathematics 3.21	Karp	348882811	CS	3.98
	Newton	1727017	Physics	3.61
Demovilli 2024/220 Mathematics 2.54	Bernoulli	2921938	Mathematics	3.21
Bernoulli 2921939 Mathematics 3.54	Bernoulli	2921939	Mathematics	3.54

Alternate Approach							
Student_ID	Name	GPA		Student_ID	Major		
328012098	Knuth	4.00		328012098	CS		
481080220	Von Neuman	3.78		481080220	CS		
238082388	Russell	3.85		481080220	Mathematics		
238001920	Einstein	2.11		238082388	Philosophy		
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2921939	Bernoulli	3.54		1727017	Physics		
				2921938	Mathematics		
				2921939	Mathematics		

