CSE 321 Discrete Structures

Winter 2008 Lecture 13 Induction and Recursion

Announcements

- Readings - Monday Recursion
 - 4.3 (5th Edition: 3.4) Midterm:
 - Friday, February 8
 - In class, closed book

 - Estimated grading weight:
 MT 12.5%, HW 50%, Final 37.5%
- · Extra Office Hour - Thursday, 5:30-6:20 pm, CSE 582
- Homework 5 is available

Highlights from Lecture 12 Mathematical Induction -P(0)- \forall k (P(k) → P(k+1)) – ∴ ∀ n P(n) Strong Induction

- P(0)
- $\forall k ((P(0) \land P(1) \land \ldots \land P(k)) \rightarrow P(k+1))$
- ∴ ∀ n P(n)

Induction Example

- A set of S integers is *non-divisible* if there is no pair of integers a, b in S where a divides b. If there is a pair of integers a, b in S, where a divides b, then S is divisible.
- Given a set S of n+1 positive integers, none exceeding 2n, show that S is divisible.
- · What is the largest subset non-divisible subset of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

If S is a set of n+1 positive integers, none exceeding 2n, then S is divisible

• Base case: n =1

· Suppose the result holds for n

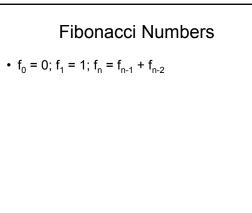
- If S is a set of n+1 positive integers, none exceeding 2n, then S is divisible
- Let T be a set of n+2 positive integers, none exceeding 2n+2. Suppose T is non-divisible.

Proof by contradiction

- Claim: $2n+1 \in T$ and $2n+2 \in T$
- Claim: n+1 ∉ T
- Let $T^* = T \{2n+1, 2n+2\} \cup \{n+1\}$
- If T is non-divisible, T* is also non-divisible

Recursive Definitions

- F(0) = 0; F(n + 1) = F(n) + 1;
- F(0) = 1; $F(n + 1) = 2 \times F(n)$;
- F(0) = 1; F(n + 1) = 2^{F(n)}



Bounding the Fibonacci Numbers

• Theorem: $2^{n/2} \le f_n \le 2^n$ for $n \ge 6$

Recursive Definitions of Sets

- Recursive definition
 - Basis step: 0 \in S
 - Recursive step: if $x \in S,$ then x + 2 \in S
 - Exclusion rule: Every element in S follows from basis steps and a finite number of recursive steps

Recursive definitions of sets

 $\begin{array}{l} \text{Basis:} \; [1, \; 1, \; 0] \in S, \; [0, \; 1, \; 1] \in S; \\ \text{Recursive:} \\ \quad \text{if } [x, \; y, \; z] \in S, \; \; \alpha \; \text{in } \; R, \; \text{then } [\alpha \; x, \; \alpha \; y, \; \alpha \; z] \in S \\ \quad \text{if } [x_1, \; y_1, \; z_1], \; [x_2, \; y_2, \; z_2] \in S \\ \quad \text{then } [x_1 + x_2, \; y_1 + y_2, \; z_1 + z_2] \end{array}$

Powers of 3

Strings

- The set Σ^{\star} of strings over the alphabet Σ is defined
 - Basis: $\lambda \in S \ (\lambda \text{ is the empty string})$
 - Recursive: if $w \in \Sigma^*$, $x \in \Sigma$, then $wx \in \Sigma^*$

Function definitions

 $\begin{array}{l} \text{Concat}(w,\,\lambda) = w \text{ for } w \in \Sigma^* \\ \text{Concat}(w_1,w_2x) = \text{Concat}(w_1,w_2)x \text{ for } w_1,\,w_2 \text{ in } \Sigma^*,\,x \in \Sigma \end{array}$