

**Problems:**

1. Prove or disprove:  $n^2 + 3n + 1$  is always prime for integer  $n > 0$ .
2. Prove that if  $n$  is an integer then  $n^2 \bmod 8$  is either 0, 1, or 4.  
Hint: Consider the different cases of  $n \bmod 4$ .
3. Section 3.4, exercise 22 [5th edition: Section 2.4, exercise 44]
4. Compute the greatest common divisor for each of the following pairs of numbers.
  - (a)  $2^1 \cdot 3^3 \cdot 5^5, 2^2 \cdot 3^3 \cdot 5^2$
  - (b)  $100!, 127$
5. Use the Euclidean algorithm to find  $\gcd(2274, 174)$ .
6. What is the rightmost digit (digit in the units place) of  $32^{631}$ ? Show your work.
7. Prove that for any prime  $p > 3$ , either  $p \equiv 1 \pmod{6}$  or  $p \equiv 5 \pmod{6}$ .
8. Section 3.5, exercise 32 [5th edition: Section 2.4, exercise 46]
9. Find an inverse of 2 modulo 17.
10. **Optional:** How many zeroes are there at the end of  $100!$  ?