

PROBLEM SET 6
Due Friday, May 18, 2007, in class

There are EIGHT problems, including an optional extra credit problem.

The exercise numbers refer to the number in Rosen's book, 6th Edition. When a different number is used in the 5th edition, that number is also mentioned. All problems are worth a total of 10 points unless mentioned otherwise.

Relevant Reading Sections: 6th Edition: 5.4, 5.5, 6.1, 6.2; 5th edition: 4.4, 4.5, 5.1, 5.2.

1. (16 points)
 - (a) Prove the Binomial Theorem using mathematical induction.
 - (b) Prove that for every integer $n \geq 1$, the number of even sized subsets of $\{1, 2, \dots, n\}$ equals the number of odd sized subsets of $\{1, 2, \dots, n\}$. (The empty set is considered to be a subset of even size.)
 - (c) Give a formula for the coefficient of x^k in the expansion of $(x + 2/x)^{100}$, where k is a nonnegative integer.
2. Section 5.4, Exercise 22. (5th Edition: Section 4.4, Exercise 22)
3. (12 points)
 - (a) Section 6.1, Exercise 28. (5th edition: Section 5.1, Exercise 28)
 - (b) Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?
 - (c) What is the probability that a five-card poker hand contains exactly two hearts and two spades?
4. Let $m \geq n$ be positive integers. How many solutions in *positive* integers x_1, x_2, \dots, x_n are there for the inequality $x_1 + x_2 + \dots + x_n \leq m$?
5. Section 6.2, Exercise 28. (5th Edition: Section 5.2, exercise 28).
6. (12 points)
 - (a) What is the conditional probability that exactly three heads appear when a fair coin is flipped five times, given that the first flip came up heads?
 - (b) Suppose we choose randomly and independently two subsets A and B from the set of all possible non-empty subsets of $\{1, 2, \dots, n\}$. What is the probability that $\min(A) = \min(B)$ (where $\min(A)$ denotes the minimum number from the set A).

7. Suppose the Boston Red Sox play a best-of-7 American League Championship Series (ALCS) with the New York Yankees (so the first team to achieve 4 victories wins the ALCS). The probability for each team to win the first game is $1/2$. However, a team that wins a game (not necessarily the first one) gets a morale boost, and so its probability to win after a victory increases to $2/3$. Similarly, after a loss, the probability of a win decreases to $1/3$.
- (a) What is the probability that the Boston Red Sox win the ALCS?
 - (b) Given that the Red Sox lose the first three games to the Yankees, what are the odds that the Red Sox make a striking comeback to win the ALCS (as it happened in 2004)?
8. * **(Bonus Problem for Extra Credit)** The 140 seats on a flight were completely booked with each of the 140 passengers having different assigned seats. The passengers entered the plane one-by-one. Unfortunately, the first passenger poured coffee over his boarding pass, couldn't read his seat number, and so sat in a uniformly random seat. Each subsequent passenger sat in their assigned seat if it was available when they entered and sat in a uniformly random unoccupied seat otherwise. What is the probability that the last passenger sat in his/her assigned seat?