CSE 321: Discrete Structures Spring 2007

PROBLEM SET 1
Due Friday, April 6, 2007, in class

Reminder: If you haven't done so already, subscribe to CSE 321 mailing list ASAP by following the link from the course webpage `http://www.cs.washington.edu/321`.

Instructions: Information on the collaboration policy and honor code in solving problem sets can be found off the course webpage — **please be sure to read it**. In a nutshell, you are allowed to collaborate with fellow students taking the class to the extent of discussing solution ideas, *provided you think about each problem on your own for at least 20 minutes.* You must write down solutions on your own, and must clearly acknowledge each person with whom you discussed the solutions. You are expected to refrain from looking up solutions from websites or other literature, and you should never be in possession of someone else's written solutions.

Please be as clear as possible in your arguments and answers. Poorly written solutions, even if more or less correct, will be penalized.

The exercise numbers refer to the number in Rosen's book, 6th Edition. When a different number is used in the 5th edition, that number is also mentioned.

1. Section 1.1, Exercise 10 (in either edition).

2. Section 1.1, Exercise 48. (Sec. 1.1, Exercise 44 in 5th Edition)

3. Section 1.1, Exercise 60. (Section 1.1, Exercise 56, in 5th edition).

4. The following two statements form the basis of the most important methods for automated theorem proving. Prove, using truth tables or otherwise, that they are tautologies.

   (a) Modus ponens: $((p \wedge (p \rightarrow q))) \rightarrow q$

   (b) Resolution: $((p \vee q) \wedge (\neg q \vee r)) \rightarrow (p \vee r)$

5. How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?

6. State in English the converse and contrapositive of each of the following implications:

   (a) If item A is pushed onto the stack before item B, then item B is popped before item A.

   (b) If the Red Sox and Mariners both make the playoffs, then either the Yankees or the Athletics will not make the playoffs.

   (c) If the input is correct and the program terminates, then the output is correct. (Be sure to use De Morgan's Law to simplify the contrapositive.)

7. Define the NAND operator, denoted $|$, as follows: $(p \mid q)$ is true when either p or q, or both are false; and it is false when p and q are true.

   (a) Show that $(p \mid q)$ and $(q \mid p)$ are equivalent, so the logical operator $|$ is commutative.

(b) Show that $p \mid (q \mid r)$ and $(p \mid q) \mid r$ are not equivalent, so the logical operator $\mid$ is *not* associative.

(c) Prove that the NAND operator is *universal* in the sense that every compound proposition is logically equivalent to a compound proposition that uses no other operator besides the NAND operator.

8. * **(Optional puzzle for your cognitive pleasure only; this doesn't bear any direct relation to current topics of discussion and need not be turned in)**
Suppose we start out with an $n \times n$ chessboard where some number k of cells are *infected*. The infection spreads in the following manner: if a square has two or more infected neighbors, then it becomes infected itself. (Neighbors are orthogonal only, so each square has at most 4 neighbors.) The process continues till the infection can spread no more. Prove that if at the end of this process all squares in the board are infected, then it must be that $k \geq n$.