CSE 321: Discrete Structures
October 29, 2007

Assignment #5
Due: **Friday**, November 9th

Reading Assignment: 6th Edition: 3.7 (just Thm. 1 and Ex. 1) and 4.2–4.3 (or, 5th Edition: 2.6 (just Thm. 1 and Ex. 1) and 3.3–3.4).

Problems:

1. Which amounts of money can be formed using just dimes and quarters? Prove your answer using a form of mathematical induction.

2. Euclid's algorithm (**show your work**).

   (a) Use Euclid's algorithm to compute $\gcd(832, 247)$.

   (b) Use the extended Euclidean algorithm to find an integer x such that $357\,x \equiv 7 \pmod{247}$.

3. Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. Any chocolate bar can be broken into two pieces along a horizontal or vertical line separating the squares (e.g. a 2×4 chocolate bar can be broken in 4 different ways: along the one horizontal line, or along one of the three vertical lines). Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into n separate squares. Use strong induction to prove your answer.

4. Give a recursive definition of

   (a) the set of odd positive integers,

   (b) the set of positive integer powers of 3,

   (c) the set of positive integers congruent to 4 modulo 5.

5. Structural induction: Show that the set S defined by $1 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$ is the set of positive integers.
   (**Hint:** You are showing that two sets A and B are equal, which requires showing that *both* $A \subseteq B$ and $B \subseteq A$.)

6. Let F_n be the nth Fibonacci number. Prove that $F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$ whenever n is a positive integer.