CSE 321: Discrete Structures Spring 2006

PROBLEM SET 7

Due Friday, May 26, 2006, in class

Reading: Sections 7.1, 7.4, 7.5

1. Suppose that n balls are tossed into b bins so that each ball is equally likely to fall into any of the bins and that the tosses are independent.
   (a) Find the probability that a particular ball lands in a specified bin.
   (b) What is the expected number of balls that land in a particular bin?
   (c) What is the expected number of balls tossed until a particular bin contains a ball?
   (d) What is the expected number of balls tossed until all bins contain a ball?

2. Let E, F be events with $P(F) \neq 0$. Prove that

$$P(E) = P(E|F)P(F) + P(E|\bar{F})P(\bar{F}) \ .$$

3. Section 7.1, Exercise 4.

4. A relation R is called *circular* if $a \ R \ b$ and $b \ R \ c$ imply that $c \ R \ a$. Show that R is reflexive and circular if and only if it is an equivalence relation.

5. Let R be a random relation on the set $A = \{a_1, a_2, \ldots, a_n\}$ selected as follows: Independently for each pair i, j, $1 \leq i \leq n$ and $1 \leq j \leq n$, include (a_i, a_j) in R with probability p. Now,
   (a) What is the probability that R is reflexive?
   (b) What is the probability that R is irreflexive? (A relation R on A is said to be *irreflexive* if for every $a \in A$, $(a, a) \notin R$.)
   (c) What is the probability that R is symmetric?
   (d) What is the probability that R is anti-symmetric?
   (e) What is the expected number of pairs $\{a_i, a_j\}$ such that $i \neq j$ and both (a_i, a_j) and (a_j, a_i) are in R.