CSE 321: Discrete Structures — Assignment #4

October 20, 2006 — Due: Friday, October 27

Reading Assignment: Rosen's text 6th Edition: sections 3.7 & 4.3 (or, 5th Edition: sections 2.6 & 3.4).

Problems:

1. 6th edition: Section 4.1, Exercise 10. (5th edition: Section 3.3, Exercise 6.)
2. 6th edition: Section 4.1, Exercise 24. (5th edition: Section 3.3, Exercise 30.)
3. Sometimes it's easier to prove a stronger statement than is apparently required. In this problem you will prove by induction that for all $n \geq 1$,
$$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} \leq \frac{3}{2}.$$
   (a) What doesn't work if you try to produce an inductive proof in which $P(n)$ is the statement that $\sum_{k=1}^{n} \frac{1}{k^3} \leq \frac{3}{2}$?
   (b) Now use induction to prove the stronger statement that for all $n \geq 1$, $\sum_{k=1}^{n} \frac{1}{k^3} \leq \frac{3}{2} - \frac{1}{2n^2}$.
4. Casting out nines: Prove that a number is divisible by 9 if and only if the sum of its digits is divisible by 9. More specifically, for a positive integer a, let digitsum(a) be the sum of the decimal digits of a. Prove by induction on the number of decimal digits in a that $a \equiv$ digitsum(a) (mod 9).
Hint: Express a as $\sum_{i=0}^{n} a_i(10)^i$ where each a_i is one of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
5. 6th edition: Section 4.3, Exercise 12. (5th edition: Section 3.4, Exercise 12.)
6. Which amounts of money can be formed using just dimes and quarters? Prove your answer using a form of mathematical induction.
7. Euclid's algorithm;
   (a) Use Euclid's algorithm to compute $\gcd(832, 247)$. (Show your work).
   (b) Use the extended Euclidean algorithm to find an integer x such that $357x \equiv 7 \pmod{247}$.
8. **Extra Credit:** Two puzzles.
   (a) Use induction to show that any rectangular checkerboard (with the usual black-white coloring) that starts with an even number of cells and then has one black cell and one white cell removed can be tiled with dominoes (2x1 cell tiles).
   (b) Consider any $n + 1$ numbers between 1 and $2n$ (inclusive). Show that some pair of them are relatively prime. Show that one is a factor of another.