CSE 321: Discrete Structures
Assignment #5
Due: Wednesday, May 11

Reading Assignment: Section 4.1-4.4 of Rosen.

Problems:

1. Section 3.3, exercise 48.

2. Recall the definition of the Fibonacci numbers: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove that f_n is even if and only if 3 divides n. (Hint: In your inductive step, consider the three cases $n \equiv 0(\text{mod}3)$, $n \equiv 1(\text{mod}3)$ and $n \equiv 2(\text{mod}3)$ separately.)

3. Prove using mathematical induction that

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2,$$

   for every positive integer $n > 1$.

4. John and Sara have a party to which they invite n other married couples. As is normal at parties, handshaking took place. Of course, none shook their own hand or their spouses hand (and not everyone shook everyone else's hand). After all the handshaking was over, John asked all the other people present including his wife Sara "how many different people's hands did you shake this evening?" Interestingly, they each gave a different answer. From the information given, deduce how many different people's hands Sara shook that evening. Prove your answer by induction on n. (Hint: try working through the solution for several small values of n before going to the general case.).

5. Solve the following counting problems. In each case show the reasoning that leads you to your answer.

   (a) A palindrome is a word that reads the same forwards and backwards. How many seven- letter palindromes can be made from the English alphabet $(a, b, \ldots, z)$?

   (b) Suppose you have n beads, each of a different color, that you need to string into a necklace? How many distinct necklaces can you make?

(A necklace flipped over remains the same and does not count as a distinct necklace.)

(c) How many different truth tables are there for propositions in n variables?

(d) How many 5 card hands from a 52 card deck have the same number of Diamonds and Hearts?

(e) How many bit strings of length 10 contain either five consecutive 0's or five consecutive 1's?