Discrete Structures

Graphs

Chapter 7, Sections 7.1 - 7.3

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Undirected Graphs

- A **simple graph** $G = (V, E)$ consists of V, a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

- A **multigraph** $G = (V, E)$ consists of a set V of vertices, a set E of edges, and a function f from E to $\{\{u, v\} \mid u, v \in V, u \neq v\}$. The edges e_1 and e_2 are called multiple or parallel edges if $f(e_1) = f(e_2)$.

- A **pseudograph** $G = (V, E)$ consists of a set V of vertices, a set E of edges, and a function f from E to $\{\{u, v\} \mid u, v \in V\}$. An edge is a loop if $f(e) = \{u, u\} = \{u\}$ for some $u \in V$.

Directed Graphs

- ◇ A **directed graph** $G = (V, E)$ consists of a set V of vertices and a set of edges E that are ordered pairs of elements of V.

- ◇ A **directed multigraph** $G = (V, E)$ consists of a set V of vertices, a set E of edges, and a function f from E to $\{(u, v) \mid u, v \in V\}$. The edges e_1 and e_2 are multiple edges if $f(e_1) = f(e_2)$.

Graph Terminology

◇ Two vertices u and v in an undirected graph G are called **adjacent** (or **neighbors**) in G if $\{u, v\}$ is an edge of G. If $e = \{u, v\}$, the edge e is called **incident with** the vertices u and v. The edge e is also said to **connect** u and v. The vertices u and v are called **endpoints** of the edges $\{u, v\}$.

◇ The **degree** of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by deg(v).

◇ **The Handshaking Theorem** : Let $G = (V, E)$ be an undirected graph with e edges. Then

$$2e = \sum_{v \in V} \text{deg}(v).$$

◇ **Theorem** : An undirected graph has an even number of vertices of odd degree.