CSE 321: Discrete Structures Spring 2004

PROBLEM SET 8

Due Friday, June 4, 2004, in class

All exercise numbers refer to the number in Rosen's book, 5th Edition.
Readings: Sections 8.2, 8.3, 8.4.

1. Prove that any (simple, undirected) graph on $n \geq 2$ vertices contains two vertices of equal degree.

2. Section 8.3, Exercise 40.

3. Section 8.4, Exercise 18. (Figure 1 appears on Page 568 of the book.)

4. Prove that if G is disconnected, then $\overline{G}$, the complement of G, is connected. (Recall that $\overline{G}$ contains all and only those edges that are absent in G.)

5. Suppose G is a (simple, undirected) graph on $2n$ vertices that contains no triangles (cycles of length 3). Prove that G has at most n^2 edges. (Hint: Induction is one approach.)

6. Consider a tournament among n players where each player plays every other player exactly once and all the $\binom{n}{2}$ games thus played result in a win/loss result. (Clearly, such a tournament can be modeled by a directed graph such that for any two distinct vertices u, v, exactly one of the directed edges (u, v) and (v, u) is present in the graph.) Prove that in any such tournament, there exists a player w who has dominated the tournament in the following sense: for every other player x, either w defeated x or defeated someone who defeated x.
   (Hint: Try for w a player who has defeated the most other players.)

7. (a) Prove that if every vertex of a (simple, undirected) graph G has degree greater than or equal to δ, then G contains a (simple) path of length δ.

   (b) * **(Bonus)** Prove that when $\delta \geq 2$, G must contain a cycle (or circuit, as it is called in the book) of length greater than or equal to $\delta + 1$.