CSE 321: Discrete Structures Spring 2004

Problem Set 3

Due Friday, April 23, 2004, in class

Instructions: Same as for Problem Set 1.

All exercise numbers refer to the number in Rosen's book, 5th Edition.

1. Prove that if you pick 10 numbers from 1 to 1000, then there is a pair of numbers such that the larger of the two is at most twice the other.
2. Section 3.3, Exercise 48.
3. Prove that there are no solutions in integers x and y to the equation $x^2 - 7y^2 = 3$.
4. A set of lines (not line segments) in the plane is said to be in general position if no two lines are parallel and no three lines intersect at a common point. Such an arrangement divides the plane into several regions. Prove that, in an arrangement of n lines in the plane in general position, the number of regions formed is exactly $\frac{n^2+n+2}{2}$.
5. Prove using mathematical induction that
$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2$$
for every positive integer $n > 1$.
6. Section 2.4, Exercise 14 (how many zeroes are there at the end of 100!).
<u>Hint</u>: Think about the unique factorization of 100! into primes.
7. Section 2.4, Exercise 46.