CSE 321: Discrete Structures
Assignment #5
November 5, 2004
Due: Friday, November 12

Reading Assignment: Read Sections 4.1-4.4.

Problems:

1. Section 3.4, exercise 12.

2. A function T is defined by $T(1) = 0$ and for all $n \geq 1$,

$$T(n+1) = \begin{cases} T(n) & \text{if } n+1 \text{ is odd} \\ T((n+1)/2) & \text{if } n+1 \text{ is even.} \end{cases}$$

   Prove that for all $n \geq 1$, $2^{T(n)} \leq n$.

3. Define a set S of strings by

Basis: The empty string $\lambda \in S$.

Constructor: If x and y are in S then $0x11x0 \in S$ and $xy \in S$.

Extremal: Nothing is in S other than the strings that can be derived by starting with the basis and applying the constructor.

   Prove that every string in S has an equal number of 0's and 1's.

4. Section 4.1, exercise 20.

5. In many sports, such as baseball, teams play a best-of-seven format in which the first team to win 4 games wins the championship. Between two teams A and B, how many different sequences of wins and losses by team A can there be in which team A wins the series. Justify your answer. (A tree diagram may be helpful.)

6. Section 4.2, exercise 32.

7. In the town of Palukaville there are three elected positions: mayor, sheriff, and dog-catcher, to be chosen from among the 70 people in town. Each person can be elected to at most one of these positions.

   (a) How many ways are there of filling all three of these positions? That is, how many different election outcomes are possible (assuming no ties)?

   (b) During the election four years ago there were only 65 people in town and someone forgot to prohibit the same person from being elected to more than one position. How many outcomes were possible then?

   (c) After a recent scandal involving a terrier, it was decided that for the next election George W. Kerry, one of the townspeople in Palukaville, will no longer eligible to be chosen as dog-catcher. In how many ways can the three positions be chosen next time?

8. **Extra Credit:** The Smithson invitational tennis tournament is a round-robin tournament (each player plays every other player once) in which each player plays at most one match per day. Suppose that there are $2n+1$ players in the tournament. How many games are played in total? What is the minimum number of days needed to run the tournament? Justify your answer. What are the corresponding answers if there only $2n$ players?