1. For each of the following functions, state whether or not it is injective, and whether or not it is surjective. Justify your answers.

(a) \( f : \mathbb{N} \rightarrow \mathbb{N} \), where \( f(n) = n^2 \).
(b) \( f : \mathbb{Z} \rightarrow \mathbb{N} \), where \( f(n) = n^2 \).
(c) \( f : \mathbb{R} \rightarrow \mathbb{R} \), where \( f(n) = 3n + 7 \).
(d) \( f : \mathbb{N} \rightarrow \mathbb{N} \), where \( f(n) = \lceil n/3 \rceil \).
(e) \( f : \mathbb{N} \rightarrow \mathbb{N} \), where \( f(n) = 3 \lceil n/3 \rceil \).
(f) \( f : \mathbb{N} \rightarrow \mathbb{N} \), where \( f(n) = \begin{cases} n + 1, & \text{if } n \text{ is even} \\ n - 1, & \text{if } n \text{ is odd} \end{cases} \).

2. Suppose you graph a function \( f : \mathbb{R} \rightarrow \mathbb{R} \). The fact that \( f \) is a function means that any straight vertical line will intersect the graph of \( f \) at exactly one point. What similar statement can you make about the graph of \( f \) if \( f \) is

(a) injective?
(b) surjective?
(c) bijective?

3. Let \( f : \mathbb{N} \rightarrow \mathbb{N} \) and \( g : \mathbb{N} \rightarrow \mathbb{N} \), where \( f(x) = x \mod 28 \) and \( g(x) = x + 1 \). What are each of the functions \( f \circ g \) and \( g \circ f \)? Either prove that these two functions are equal, or give a counterexample proving that they are unequal.

4. Draw the graph of the function \( f : \mathbb{R} \rightarrow \mathbb{R} \), where \( f(x) = \lfloor x/4 \rfloor \). Be sure your graph extends into both positive and negative values of \( x \).


6. Section 2.3, exercise 12. Justify your answer. The function \( n! \) is defined on page 85. (Hint: Think about the unique factorization of 100! into primes. What about this factorization determines the number of zeros at the end of the decimal representation of 100! ?)