CSE 321: Discrete Structures
Assignment #8
Due Wednesday, December 3

Reading: Rosen, Sections 7.1,7.4, 7.5 and Chapter 8 (In 4th edition, Sections 6.1, 6.4 and 6.5, chapter 7).

1. Determine whether the following relations R on the set of all people are reflexive, symmetric, antisymmetric and/or transitive where $(a, b) \in R$ if and only if
   - a is taller than b
   - a and b were born on the same day
   - a has the same first name as b
   - a and b have a common grandparent.
2. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.
3. These questions refer to the relation defined in the previous question.
   - What is the equivalence class of $(1, 2)$ with respect to the equivalence relation of the previous question?
   - Give an interpretation of the equivalence classes for the equivalence relation R of the previous question. The answer should show that there is a one-to-one correspondence between the equivalence classes of R and the elements of a very familiar mathematical set.
4. A relation R is called *circular* if $a\ R\ b$ and $b\ R\ c$ imply that $c\ R\ a$. Show that R is reflexive and circular if and only if it is an equivalence relation.
5. Let R be a random relation on a set $A = \{a_1, a_2, \ldots, a_n\}$ selected as follows: Independently, for each pair i, j, $1 \leq i \leq n$ and $1 \leq j \leq n$, (a_i, a_j) is included in R with probability $p_i \cdot p_j$. What is the probability that R is reflexive? What is the expected number of pairs (a_i, a_i) in R?
6. Find the number of paths of length n between any two adjacent vertices in $K_{3,3}$ for n equals 2, 3, 4 and 5. See Example 11 on page 550 (page 450 in 4th edition) Note that paths need not be simple. Use counting techniques rather than trying to list all paths.