CSE 321: Discrete Structures
Assignment #7
Due Wednesday, November 26

Reading: Rosen, Sections 7.1, 7.4, 7.5, and chapter 8. In the 4th edition, these are 6.1, 6.4-6.5 and chapter 7).

1. A deck of 10 cards, each bearing a distinct number from 1 to 10, is shuffled to mix the cards thoroughly, so that each order is equally likely. What is the probability that the top three cards are in sorted (increasing) order?

2. Suppose that A and B are events in a probability space, and that $Pr(A) = 0.5$, $Pr(B) = 0.2$ and $Pr(A \cup B) = 0.6$. What is $Pr(A \cap B)$?

3. Suppose that each of the students in a 100 person class is assigned uniformly and independently to one of four quiz sections. What is the probability that all six students named "David" are assigned to the same section?

4. Eight men and seven women, all single, happen randomly to have purchased single seats in the same 15-seat row of a theatre. What is the probability that the first two seats contain a (legally) marriageable couple?

5. A fair coin is flipped n times. What is the probability that all the heads occur at the end of the sequence?

6. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

7. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?

8. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has

   (a) exactly three boys?
   (b) at least one boy?
   (c) at least one girl?
   (d) all children of the same sex?

9. Suppose a 6-sided fair dice is rolled. Let the random variable X be the value showing. What is the expectation of X? Suppose two fair 6-sided dice are rolled independently. Let Y be the random variable which is the sum of the two values showing. What is the expectation of Y? Let Z be the random variable which is the maximum of the two values showing. What is the expectation of Z?

10. Suppose that a fair coin is tossed 100 times. Let X be the random variable which is the number of flips i in which the coin takes on the same value in both flip i and $i+1$. What is the expected value of X? (So for example in the sequence HHHH, X is 3, because the coin takes on the same value in positions 1 and 2, 2 and 3, and 3 and 4. In the sequence THHHTT, the X is also 3 because the coin takes on the same value in positions 2 and 3, 3 and 4, and 5 and 6.)