Sample Proof

CSE 321

January 20, 2001

Proof that $(A \cup B = B) \Rightarrow (A \subseteq B)$:

1. $(A \cup B = B) \Rightarrow ((x \in A \vee x \in B) \Leftrightarrow (x \in B))$ (def. of subset)
2. $\Leftrightarrow ((x \in A \vee x \in B) \Rightarrow x \in B) \wedge (x \in B \Rightarrow (x \in A \wedge x \in B))$ (def. of iff)
3. $\Leftrightarrow (\neg(x \in A \vee x \in B) \vee (x \in B)) \wedge (\neg(x \in B) \vee (x \in A \vee x \in B))$ (since $p \Rightarrow q \Leftrightarrow \neg p \vee q$)
4. $\Leftrightarrow ((x \notin A \wedge x \notin B) \vee (x \in B)) \wedge (x \notin B \vee (x \in A \wedge x \in B))$ (Demorgan's)
5. $\Leftrightarrow ((x \notin A \wedge x \in B) \wedge (x \notin B \vee x \in B)) \wedge (x \notin B \vee (x \in A \vee x \in B))$ (Distributive)
6. $\Leftrightarrow ((x \notin A \wedge x \in B) \wedge (x \notin B \vee x \in B)) \wedge ((x \notin B \vee x \in A) \vee (x \notin B \vee x \in B))$ (Distributive)
7. $\Leftrightarrow ((x \notin A \vee x \in B) \vee T) \wedge ((x \notin B \vee x \in A) \vee T)$ (since $p \vee \neg p \Leftrightarrow T$)
8. $\Leftrightarrow ((x \notin A \vee x \in B) \wedge T)$ (since $p \vee T \Leftrightarrow T$)
9. $\Leftrightarrow (\neg(x \in A) \vee (x \in B))$ (since $p \wedge T \Leftrightarrow p$)
10. $\Leftrightarrow (x \in A \Rightarrow x \in B)$ (since $p \Rightarrow q \Leftrightarrow \neg p \vee q$)
11. $\Leftrightarrow (A \subseteq B)$ (def. of subset)