Inference Rules in FOL

Premises:

1. If x is a parent of y, then y is older than x
2. If x is the mother of y, then x is a parent of y
3. Lulu is the mother of Fifi

Conclusion:

Lulu is older than Fifi

Premises in first-order logic:

1. $\forall x \forall y.Parent(x,y) \Rightarrow Older(x, y)$
2. $\forall x \forall y.Mother(x,y) \Rightarrow Parent(x, y)$
3. $Mother(Lulu, Fifi)$

Inference Rules in FOL (cont.)

Previous inference easier with strengthened rule:

Generalized Modus Ponens

$$\frac{\alpha_1[\underline{x}/\underline{k}],\ldots,\alpha_n[\underline{x}/\underline{k}],\ \forall x_1\ldots\forall x_m.(\alpha_1\wedge\ldots\wedge\alpha_n) \Rightarrow \beta}{\beta[\underline{x}/\underline{k}]}$$

$\underline{x}/\underline{k}$: $\{x_1/k_1,\ldots,x_m/k_m\}$

A proof of Older(Lulu, Fifi) using GMP:

1. Mother(Lulu, Fifi)	given
2. Alive(Lulu)	given
3. $\forall x \forall y.$Mother$(x,y) \Rightarrow$ Parent(x, y)	given
4. $\forall x \forall y.$(Parent$(x,y) \wedge$ Alive$(x)) \Rightarrow$ Older(x, y)	given
5. Parent(Lulu, Fifi)	1,3, GMP
6. Older(Lulu, Fifi)	5,2,4, GMP

This use of GMP is called forward-chaining

Inference Rules in FOL (cont.)

Another style of proof is to "reason backward" (backward-chaining):

> Start with a goal (to be proved) & then derive new sub-goals until we have sub-goals known to be true
>
> $\longrightarrow$ similar to problem reduction

Backward-chaining proof of Older(Lulu, Fifi) from premices:

1. Mother(Lulu, Fifi)
2. Alive(Lulu)
3. $\forall x \forall y. Mother(x,y) \Rightarrow Parent(x, y)$
4. $\forall x \forall y. (Parent(x,y) \wedge Alive(x)) \Rightarrow Older(x, y)$

Goal: (i) Older(Lulu, Fifi)

Match (i) against RHS of (4)

Subgoals: (ii) Parent(Lulu, Fifi) (iii) Alive(Lulu)

Match (iii) against (2) True

Match (ii) against RHS of (3)

Subgoal: (iv) Mother(Lulu, Fifi)

Match (iv) against (1)