Discrete Structures — Anna Karlin
CSE 321, Spring 2000

Homework #7
Due Wednesday, May 24

Reading: Rosen, Sections 6.1, 6.5, 7.1–7.2

1. A deck of 10 cards, each bearing a distinct number from 1 to 10, is shuffled to mix the cards thoroughly, so that each order is equally likely. What is the probability that the top three cards are in sorted (increasing) order?
2. Suppose that A and B are events in a probability space, and that $Pr(A) = 0.5$, $Pr(B) = 0.2$ and $Pr(A \cup B) = 0.6$. What is $Pr(A \cap B)$?
3. Suppose that each of the students in a 100 person class is assigned uniformly and independently to one of four quiz sections. What is the probability that all six students named "David" are assigned to the same section?
4. Eight men and seven women, all single, happen randomly to have purchased single seats in the same 15-seat row of a theatre. What is the probability that the first two seats contain a (legally) marriageable couple?
5. A fair coin is flipped n times. What is the probability that all the heads occur at the end of the sequence?
6. Rosen, Section 4.5, problem 16.
7. Rosen, Section 4.5, problem 18.
8. Rosen, Section 4.5, problem 20.
9. Suppose a 6-sided dice is rolled. What is the expectation of the value showing? Suppose two 6-sided dice are rolled. What is the expectation of the value showing? What is the expectation of the maximum of the two values showing?
10. Suppose that a fair coin is tossed 100 times. What is the expected number of flips i in which the coin takes on the same value in both flip i and flip i+1? (So for example in the sequence HHHH, the answer is 3, because the coin takes on the same value in positions 1 and 2, 2 and 3, and 3 and 4. In the sequence THHHTT, the answer is also 3 because the coin takes on the same value in positions 2 and 3, 3 and 4, and 5 and 6.)
11. **Extra credit** You are taking an exam with 6 questions. You haven't studied very much, so the probability that you can answer a given question is only 1/3. However, your neighbor, the best student in the class, can answer a given question with probability 9/10. So, you can (should?) do the exam by yourself, or you can try to cheat... For every question on which you try to cheat, there is a probability 1/6 that you will be caught. If you are caught, you get zero for the whole exam. However, if you are not caught, you get the answer from your neighbor, which is likely to be better than your own. What strategy will maximise your expected grade for this exam? That is, should you try to cheat? How many times?