CSE 312 Section 3

Conditional Probability

Administrivia

Announcements & Reminders

- HW1
  - Grades released on gradescope – check your submission to read comments
  - Regrade requests open ~24 hours after grades are released and close after a week

- HW2
  - Was due yesterday, Wednesday 01/21 @ 11:59pm
  - Late deadline Saturday 01/24 @ 11:59pm (max of 3 late days per problem)

- HW3
  - Released on the course website
  - Due Wednesday 01/28 @ 11:59pm

Review

Any lingering questions from this last week?

Each week in section, we'll be reviewing the main concepts from this week and putting them into action by going through some practice problems together. But before we get into that review, we'll try to start off each section with some time for you to ask questions. Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

Quiz

Problem 5 – Parallel Systems

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

b) If the system is functioning, what is the probability that component 1 is working?

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

Work on this problem with the people around you, and then we'll go over it together!

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

Let C_i be the event component i is working, and F be the event that the system is functioning. For the system to function, it is sufficient for any component to be working. This means that the only case in which the system does not function is when none of the components work. We can then use complementing to compute $\mathbb{P}(F)$, knowing that $\mathbb{P}(C_i) = p$. We get:

$$\mathbb{P}(F) = 1 - \mathbb{P}(F^C) = 1 - \mathbb{P}\left(\bigcap_{i=1}^{n} C_i^C\right) = 1 - \prod_{i=1}^{n} \mathbb{P}(C_i^C)$$

$$= 1 - \prod_{i=1}^{n} \left(1 - \mathbb{P}(C_i)\right) = 1 - \prod_{i=1}^{n} (1 - p) = 1 - (1 - p)^n$$

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

b) If the system is functioning, what is the probability that component 1 is working?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

b) If the system is functioning, what is the probability that component 1 is working?

We know that for the system to function only one component needs to be working, so for all i, we have $\mathbb{P}(F|C_i) = 1$. Using Bayes Theorem, we get

$$\mathbb{P}(C_1|F) = \frac{\mathbb{P}(F|C_1)\mathbb{P}(C_1)}{\mathbb{P}(F)} = \frac{1 \cdot p}{1-(1-p)^n} = \frac{p}{1-(1-p)^n}$$

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

$$\mathbb{P}(C_1|C_2, F) = \mathbb{P}(C_1|C_2) = \mathbb{P}(C_1) = p$$

The first equality holds because knowing C_2 and F is just as good as knowing C_2 (since if C_2 happens, F does too), and the second equality holds because the components working are independent of each other.

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

More formally, we can use the definition of conditional probability along with a careful application of the chain rule to get the same result. We start with the following expression:

$$\mathbb{P}(C_1|C_2,F) = \frac{\mathbb{P}(C_1,C_2,F)}{\mathbb{P}(C_2,F)} = \frac{\mathbb{P}(F|C_1,C_2)\mathbb{P}(C_1|C_2)\mathbb{P}(C_2)}{\mathbb{P}(F|C_2)\mathbb{P}(C_2)}$$

We note that the system is guaranteed to work if any one component is working, so $\mathbb{P}(F|C_1,C_2) = \mathbb{P}(F|C_2) = 1$. We also note that components work independently of each other, hence $\mathbb{P}(C_1|C_2) = \mathbb{P}(C_1)$. With that in mind, we can rewrite our expression so that:

$$\mathbb{P}(C_1|C_2,F) = \frac{1 \cdot \mathbb{P}(C_1)\mathbb{P}(C_2)}{1 \cdot \mathbb{P}(C_2)} = \mathbb{P}(C_1) = p$$

Problem 9 – Dependent Dice Duo

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that

$$\mathbb{P}(E \cap F | G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$$

Work on this problem with the people around you, and then we'll go over it together!

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that $\mathbb{P}(E \cap F|G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that $\mathbb{P}(E \cap F|G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

$$\mathbb{P}(E|G) = \mathbb{P}(D_1 = 1|D_1 + D_2 = 7) = \frac{1}{6}$$

$$\mathbb{P}(F|G) = \mathbb{P}(D_2 = 6|D_1 + D_2 = 7) = \frac{1}{6}$$

$$\mathbb{P}(E \cap F|G) = \mathbb{P}(D_1 = 1 \cap D_2 = 6|D_1 + D_2 = 7) = \frac{1}{6}$$

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that $\mathbb{P}(E \cap F|G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

When we condition on G, our sample space Ω becomes {(1,6), (2,5), (3,4), (4,3), (2,5), (6,1)}, where the first number in the pair is the D_1 outcome and the second number is the D_2 outcome.

From here we can see that $P(E|G) = P(D_1 = 1 \mid D_1 + D_2 = 7) = 1/6$ as (1,6) is 1 of the 6 possible rolls that sum to 7 and each roll is equally likely.

We also see that $P(F|G) = P(D_2 = 6 \mid D_1 + D_2 = 7) = 1/6$ and $P(E \cap F|G) = P(D_1 = 1 \cap D_2 = 6 \mid D_1 + D_2 = 7) = 1/6$ using similar reasoning. Now we have that $P(E|G) * P(F|G) = 1/36$. Notice that $1/36 \neq 1/6$ so we have shown that independence can be "broken" by conditioning.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?