## Union Bound

For any events $E, F$
$\mathbb{P}(\boldsymbol{E} \cup \boldsymbol{F}) \leq \mathbb{P}(\boldsymbol{E}) + \mathbb{P}(\boldsymbol{F})$

## (Multiplicative) Chernoff Bound

**Let $X_1, X_2, \dots, X_n$ be *independent* Bernoulli random variables.**
***Let* $X = \sum X_i$, and $\mu = \mathbb{E}[X]$. For any $0 \leq \delta \leq 1$**

$$\mathbb{P}(X \geq (1+\delta)\mu) \leq \exp\left(-\frac{\delta^2 \mu}{3}\right) \text{ and } \mathbb{P}(X \leq (1-\delta)\mu) \leq \exp\left(-\frac{\delta^2 \mu}{2}\right)$$

44

# Example

Suppose you flip a coin independently 10 times, and you see

HTTTHHTHHH

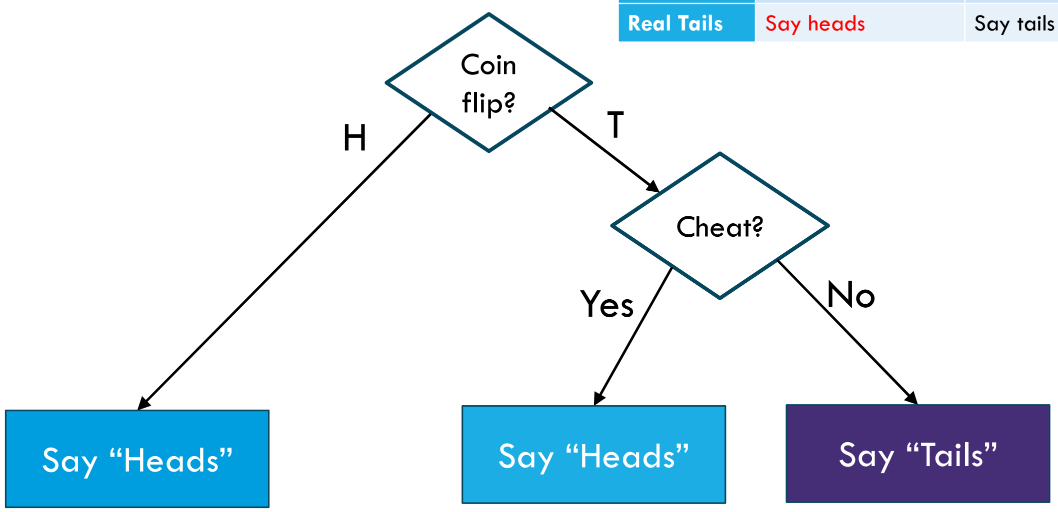
What is your estimate of the probability the coin comes up heads?

A. Something less than 0.5
B. 0.5
C. Something between 0.5 and 0.6
D. 0.6
E. Something more than 0.6

35

# Warner's Protocol

| Result | Yes (cheated) | No (never cheated) |
|---|---|---|
| Real Heads | Say heads | Say heads |
| Real Tails | Say heads | Say tails |




16

# But will it be accurate?

But we've lost our data haven't we? People answered a different question. Can we still estimate how many people cheated?

Suppose you asked 100 people the "heads/tails" question, and 60 people said "heads." What do you predict would be the number of people who cheated on a partner?

Can you generalize your idea for $n$ people polled, and $X$ the number of people that said "heads"?

19