Where are we?

A random variable is a numerical summary of the outcome of an experiment.

$\mathbb{E}[X]$ is the weighted average of possibilities of X.

For <u>all</u> rv's $\mathbb{E}[X+Y]=\mathbb{E}[X]+\mathbb{E}[Y]$.

Indicator rv's are a great trick to simplify expectation computations.

$\mathrm{Var}(X)=\mathbb{E}[(X-\mathbb{E}[X])^2]$ measures how "spread out" a rv is.

For <u>independent</u> rv's:

$\mathrm{Var}(X+Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)$

$\mathbb{E}[XY]=\mathbb{E}[X]\mathbb{E}[Y]$

2

Shifting a random variable

For any random variable $\mathbf{X}$, and any constants $\mathbf{a}, \mathbf{b}$:
$\mathbb{E}[\boldsymbol{aX}+\boldsymbol{b}]=\boldsymbol{a}\mathbb{E}[\boldsymbol{X}]+\boldsymbol{b}$

For any random variable $\mathbf{X}$, and any constants $\mathbf{a}, \mathbf{b}$:
$\mathbf{Var}(\boldsymbol{aX}+\boldsymbol{b})=\boldsymbol{a}^{\mathbf{2}}\mathbf{Var}(\mathbf{X})$

66

Zoo!

$X\sim\mathrm{Unif}(a,b)$	$X\sim\mathrm{Ber}(p)$	$X\sim\mathrm{Bin}(n,p)$	$X\sim\mathrm{Geo}(p)$
$p_X(k)=\frac{1}{b-a+1}$ $\mathbb{E}[X]=\frac{a+b}{2}$ $\mathrm{Var}(X)=\frac{(b-a)(b-a+2)}{12}$	$p_X(0)=1-p;$ $p_X(1)=p$ $\mathbb{E}[X]=p$ $\mathrm{Var}(X)=p(1-p)$	$p_X(k)=\binom{n}{k}p^k(1-p)^{n-k}$ $\mathbb{E}[X]=np$ $\mathrm{Var}(X)=np(1-p)$	$p_X(k)=(1-p)^{k-1}p$ $\mathbb{E}[X]=\frac{1}{p}$ $\mathrm{Var}(X)=\frac{1-p}{p^2}$

$X\sim\mathrm{NegBin}(r,p)$	$X\sim\mathrm{HypGeo}(N,K,n)$	$X\sim\mathrm{Poi}(\lambda)$
$p_X(k)=\binom{k-1}{r-1}p^r(1-p)^{k-r}$ $\mathbb{E}[X]=\frac{r}{p}$ $\mathrm{Var}(X)=\frac{r(1-p)}{p^2}$	$p_X(k)=\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$ $\mathbb{E}[X]=n\frac{K}{N}$ $\mathrm{Var}(X)=\frac{K(N-K)(N-n)}{N^2(N-1)}$	$p_X(k)=\frac{\lambda^k e^{-\lambda}}{k!}$ $\mathbb{E}[X]=\lambda$ $\mathrm{Var}(X)=\lambda$

67

Formally…

Let X be the total number of flips needed, Y be the flips after the second.

$\mathbb{P}(Y=k|X\geq 3)=?$

…

Which is $p_X(k)$.

28