# Try it yourself

Let $X$ be the result shown on a fair die. What is $\mathbb{E}[X]$?

Let $Y$ be the sum of two (independent) fair die rolls. What is $\mathbb{E}[Y]$?

| Expectation |
| --- |
| **The "expectation" (or "expected value") of a random variable $X$ is:** $$\mathbb{E}[X] = \sum_k k \cdot \mathbb{P}(X = k)$$ |

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# Variance

| Variance |
| --- |
| **The variance of a random variable $X$ is** $$\mathrm{Var}(X) = \sum_\omega \mathbb{P}(\omega) \cdot (X(\omega) - \mathbb{E}[X])^2 = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$ |

The first two forms are the definition. The last one is an algebra trick.

| **If $X$ and $Y$ are independent, then** $$\mathbf{Var}(X + Y) = \mathbf{Var}(X) + \mathbf{Var}(Y)$$ |
| --- |

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# What does Independence give you?

$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$

$\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$

What does $XY$ mean? I rolled two dice, let $X$ be the red die, $Y$ the blue die. $XY$ is the random variable that tells you the product of the two dice.

That's a function that takes in an outcome and gives you a number back...so a random variable!! (Same for $X + Y$).

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# Independence of Random Variables

That's for events...what about random variables?

**Independence (of random variables)**

$X$ **and** $Y$ **are independent if for all** $k, \ell$
$\mathbb{P}(X = k, Y = \ell) = \mathbb{P}(X = k)\mathbb{P}(Y = \ell)$

We'll often use commas instead of ∩ symbol.

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