Two descriptions

PROBABILITY MASS FUNCTION

Defined for all $\mathbb{R}$ inputs.

Usually has "0 otherwise" as an extra case.

$\sum_x p_X(x) = 1$

$0 \le p_X(x) \le 1$

$\sum_{z:z\le x} p_X(z) = F_X(x)$

CUMULATIVE DISTRIBUTION FUNCTION

Defined for all $\mathbb{R}$ inputs.

Often has "0 otherwise" and 1 otherwise" extra cases

Non-decreasing function

$0 \le F_X(x) \le 1$

$\lim_{x\to-\infty} F_X(x) = 0$

$\lim_{x\to\infty} F_X(x) = 1$

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Try It Yourself

There are 20 balls, numbered 1,2,…,20 in an urn.

You'll draw out a size-three subset. (i.e. without replacement)

$\Omega = \{$size three subsets of $\{1, \dots, 20\}\}$, $\mathbb{P}()$ is uniform measure.

Let X be the largest value among the three balls.

If outcome is $\{4,2,10\}$ then $X = 10$.

Write down the PMF of X; Write down the CDF of X.

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Random Variable

$X: \Omega \to \mathbb{R}$ **is a random variable**
$X(\omega)$ **is the summary of the outcome** ω

Expectation

The "expectation" (or "expected value") of a random variable X **is:**

$$\mathbb{E}[X] = \sum_{k} k \cdot \mathbb{P}(X = k)$$

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Try it yourself

Let X be the result shown on a fair die. What is $\mathbb{E}[X]$?

Let Y be the sum of two (independent) fair die rolls. What is $\mathbb{E}[Y]$?

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