# Conditioning (Wonka Bars)

Let $S$ be the event that the Scale alerts you

Let $G$ be the event your bar has a Golden ticket.

What conditional probabilities are each of these?

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.

If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

You pick up a bar and it alerts, what is the probability you have a golden ticket?

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# LTP and Bayes

## Law of Total Probability

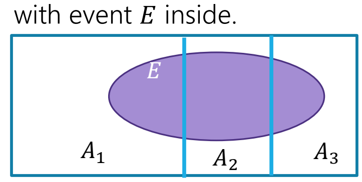
Let $A_1, A_2, \ldots, A_k$ be a **partition** of $\Omega$.
**For any event $E$,**

$$\mathbb{P}(\boldsymbol{E}) = \sum_{\textbf{all } i} \mathbb{P}(\boldsymbol{E}|\boldsymbol{A_i})\mathbb{P}(\boldsymbol{A_i})$$

## Bayes' Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

$\Omega$, split into partition $A_1, A_2, A_3$ with event $E$ inside.

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# Independence

**Independence**

**Two events $A, B$ are independent if**
$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

You'll sometimes see this called "statistical independence" to emphasize that we're talking about probabilities (not, say, physical interactions).

If $A, B$ both have non-zero probability then

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \Leftrightarrow \mathbb{P}(A|B) = \mathbb{P}(A) \Leftrightarrow \mathbb{P}(B|A) = \mathbb{P}(B)$$

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# Conditional Independence Example

You have two coins. Coin $A$ is fair, coin $B$ comes up heads with probability $0.85$.

You will roll a (fair) die, if the result is odd flip coin $A$ twice (independently); if the result is even flip coin $B$ twice (independently)

Let $C_1$ be the event "the first flip is heads", $C_2$ be the event "the second flip is heads", $O$ be the event "the die was odd"

Are $C_1$ and $C_2$ independent? Are they independent conditioned on $O$?

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