Conditioning Practice

Red die 6 conditioned on sum 7

Red die 6 conditioned on sum 9

Sum 7 conditioned on red die 6

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
D1=1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1.6)
D1=2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
D1=3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
D1=4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
D1=5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
D1=6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

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Conditional Probability

Conditional Probability

For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on $\boldsymbol{B}$" is

$$\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$$

Just like with the formal definition of probability, this is pretty abstract.

It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has not happened) – $\mathbb{P}(A|B)$ is undefined when $\mathbb{P}(B) = 0$.

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Another Example

Suppose you shuffle a deck of cards so any arrangement is equally likely. What is the probability that the top two cards have the same value?

Sample Space

Probability Measure

Event

Probability

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LTP and Bayes

Law of Total Probability

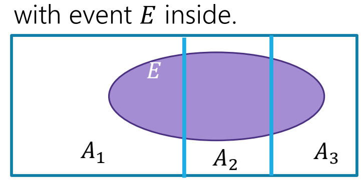
Let $A_1, A_2, \ldots, A_k$ be a **partition** of Ω.
For any event E,

$$\mathbb{P}(\boldsymbol{E}) = \sum_{\text{all } i} \mathbb{P}(\boldsymbol{E}|\boldsymbol{A_i})\mathbb{P}(\boldsymbol{A_i})$$

Bayes' Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Ω, split into partition A_1, A_2, A_3 with event E inside.



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