CSE 312 : Autumn 2025 Quiz 4 Retake Form 1 Solutions

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1. Central Limit Theorem [10 points]

Suppose you have a fair, 5-sided die, with sides numbered 1 through 5. Let X be the sum of 40 independent rolls of the die.

(a) Calculate the expected value of X. [2 points]

Solution:

Let $X_i \sim \text{Unif}(1, 5)$ (discrete) be the outcome of the ith roll. We have $\mathbb{E}[X_i] = \frac{1+5}{2} = 3$, and $X = \sum_{i=1}^{40} X_i$, so by LoE,

$$\mathbb{E}[X] = \sum_{i=1}^{40} \mathbb{E}[X_i] = 40(3) = 120$$

(b) Calculate the variance of X. [2 points]

Solution:

Using X_i as defined in the solution for (a), $\text{Var}(X_i) = \frac{(5-1)(5-1+2)}{12} = 2$.
Since variances add for independent RVs,

$$\text{Var}(X) = \sum_{i=1}^{40} \text{Var}(X_i) = 40(2) = 80$$

(c) Using the Central Limit Theorem, estimate the probability that the sum of our dice rolls is between 80 and 150 inclusive. Write your answer in terms of $\Phi(\cdot)$, the CDF of a $\mathcal{N}(0, 1)$ random variable.
In applying the CLT, **apply a continuity correction** if and only if it is usually appropriate for approximating a variable like X.
You may also write your answer in terms of a and b, your answers from parts (a) and (b) respectively. [6 points]

Solution:

We are solving for $\mathbb{P}(80 \le X \le 150)$. By CLT, we can approximate X using $Y \sim \mathcal{N}(a, b)$, where $Z = \frac{Y-a}{\sqrt{b}} \sim \mathcal{N}(0, 1)$. Then,

$$\begin{aligned}
\mathbb{P}(80 \le X \le 150) &= \mathbb{P}(79.5 \le X \le 150.5) && \text{continuity correction} \\
&\approx \mathbb{P}(79.5 \le Y \le 150.5) && X \approx Y \text{ by CLT} \\
&= \mathbb{P}\left(\frac{79.5 - a}{\sqrt{b}} \le Z \le \frac{150.5 - a}{\sqrt{b}}\right) && \text{standard.} \\
&= \mathbb{P}\left(Z \le \frac{150.5 - a}{\sqrt{b}}\right) - \mathbb{P}\left(Z \le \frac{79.5 - a}{\sqrt{b}}\right) \\
&= \Phi\left(\frac{150.5 - a}{\sqrt{b}}\right) - \Phi\left(\frac{79.5 - a}{\sqrt{b}}\right) && \text{def. of } \Phi(\cdot)
\end{aligned}$$

where $a = 120$ and $b = 80$.

2. Heads or Tails? [12 points]

Suppose you flip a biased coin 200 times (independent of each other), where each flip comes up tails with probability 0.6. Let Y be the total number of tails.
Note that $\mathbb{E}[Y] = 120$.

(a) Calculate the variance of Y. [2 points]

Solution:

Var$(Y) = 200(0.6)(1 - 0.6) = 48$ from the zoo.

(b) Using Markov's inequality, give a bound on the probability that we see at least 170 tails. [5 points]

Solution:

By Markov,

$$\mathbb{P}(Y \geq 170) \leq \frac{E[Y]}{170} = \frac{120}{170}$$

(c) Using Chebyshev's inequality, give a bound on the probability that we see between 106 and 134 tails. You may write your answer in terms of a, your answer from part (a). [5 points]

Solution:

We are solving for $\mathbb{P}(106 \leq Y \leq 134) = \mathbb{P}(|Y - 120| \leq 14)$, since Y being between 106 and 134 means it is at most 14 away from the expectation. By the complement rule, this is equal to $1 - \mathbb{P}(|Y - 120| \geq 14)$. By Chebyshev,

$$\mathbb{P}(|Y - 120| \geq 14) \leq \frac{\text{Var}(Y)}{14^2} = \frac{a}{14^2}$$

so

$$\mathbb{P}(|Y - 120| \leq 14) \geq 1 - \frac{a}{14^2}$$

for $a = 48$.

3. Multiple Choice [3 points]

(a) Let random variables X, Y be as defined in problems 1 and 2 respectively; recall that X is the sum of 40 independent rolls of a fair 5-sided die, and Y is the number of tails in 200 independent flips of a biased coin that comes up tails with probability 0.6.

Suppose we want to bound $\mathbb{P}(X \geq 180)$ and $\mathbb{P}(Y \geq 180)$. Can we (directly) apply the Chernoff bound? [3 points]

- ◯ Yes, for $\mathbb{P}(X \geq 180)$ only
- ◯ Yes, for $\mathbb{P}(Y \geq 180)$ only
- ◯ Yes, for both $\mathbb{P}(X \geq 180)$ and $\mathbb{P}(Y \geq 180)$
- ◯ No, we cannot use Chernoff to bound either expression

Solution:

2nd option: $\mathbb{P}(Y \geq 180)$ only, since X is a sum of uniform RVs; Y is a sum of independent Bernoulli RVs.