CSE 312 : Autumn 2025 Quiz 4 Form 1 Solutions

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1. Squirrels! [12 points]

You are spotted by a bunch of UW's campus squirrels while on your way to lecture! There are 70 squirrels hidden throughout campus, and each squirrel comes up to visit you (and beg for food) independently with probability 0.3. Let X be the total number of squirrels that visit you.

(a) Calculate the expected value of X. [3 points]

Solution:

$X \sim \mathsf{Bin}(70, 0.3)$, so $\mathbb{E}[X] = 70(0.3) = 21$

(b) Calculate the variance of X. [3 points]

Solution:

$\text{Var}(X) = 70(0.3)(1-0.3) = 70(0.3)(0.7) = 14.7$

(c) Using the Central Limit Theorem to estimate the probability that you are visited by at least 20 squirrels. Write your answer in terms of $\Phi(\cdot)$, the CDF of a $\mathcal{N}(0,1)$ random variable.
In applying the CLT, **apply a continuity correction** if and only if it is normally appropriate for approximating a variable like X.
You may also write your answer in terms of a and b, your answers from parts (a) and (b) respectively. [6 points]

Solution:

We are solving for $\mathbb{P}(X \geq 20)$. By CLT, we can approximate X using $Y \sim \mathcal{N}(a, b)$, where $Z = \frac{Y-a}{\sqrt{b}} \sim \mathcal{N}(0,1)$. Then,

$$\begin{aligned}
\mathbb{P}(X \geq 20) &= \mathbb{P}(X \geq 19.5) && \text{continuity correction} \\
&\approx \mathbb{P}(Y \geq 19.5) && X \approx Y \text{ by CLT} \\
&= \mathbb{P}\left(Z \geq \frac{19.5-a}{\sqrt{b}}\right) && \text{standardizing} \\
&= 1 - \mathbb{P}\left(Z \leq \frac{19.5-a}{\sqrt{b}}\right) && \text{complement} \\
&= \boxed{1 - \Phi\left(\frac{19.5-a}{\sqrt{b}}\right)} && \text{def. of } \Phi(\cdot)
\end{aligned}$$

$= 1 - \Phi\left(\frac{19.5-21}{\sqrt{14.7}}\right)$ with our values of a, b.

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2. More Squirrels [10 points]

This is exactly the same setup as problem 1.
You are spotted by a bunch of UW's campus squirrels while on your way to lecture! There are 70 squirrels hidden throughout campus, and each squirrel comes up to visit you (and beg for food) independently with probability 0.3. Let X be the total number of squirrels that visit you.

For this problem, let a and b be the expectation and variance of X, i.e. correct answers from **problem 1, parts (a) and (b)** respectively.

(a) Use Markov's inequality to bound the probability that we are visited by at most 48 squirrels.
You may write your answer in terms of a. [5 points]

Solution:

We are looking for $\mathbb{P}(X \leq 48)$. This is the same as $1 - \mathbb{P}(X \geq 49)$.
Since X is non-negative random variable, we can use Markov bound to first compute $\mathbb{P}(X \geq 49)$

$$\mathbb{P}(X \geq 49) \leq \frac{a}{49}$$

Then we have

$$\begin{aligned}\mathbb{P}(X \leq 48) &= 1 - \mathbb{P}(X \geq 49) \\ &\geq 1 - \frac{a}{49}\end{aligned}$$

(b) Use Chebyshev's inequality to bound the probability that we are visited by at least 30 squirrels. You can assume that $a < 30$.
You may write your answer in terms of a and b. [5 points]

Solution:

We are looking to upper bound $\mathbb{P}(X \geq 30)$. By Chebyshev's Inequality, we have

$$\begin{aligned}\mathbb{P}(X \geq 30) &= \mathbb{P}(X - a \geq 30 - a) \\ &\leq \mathbb{P}(|X - a| \geq 30 - a) \\ &\leq \frac{b}{(30-a)^2}\end{aligned}$$

3. Multiple Choice

(a) Let A, B, C be events. Then, we have $\mathbb{P}(A \cup B \cup C) > \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C)$. [3 points]

◯ Always
◯ Sometimes
◯ Never

Solution:

Never, by union bound (or PIE)