CSE 312 : Autumn 2025 Quiz 3 Retake Form 1 Solutions

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1. Short Answer Questions [25 points]

(a) Let X be a random variable with PDF

$$f_X(x) = \begin{cases} \frac{x^2}{c} & 1 \le x \le 7 \\ 0 & \text{otherwise} \end{cases}$$

What value of c would make this a valid pdf?
For this part, an answer is considered fully simplified if someone with a calculator but no knowledge of calculus could get an exact number. [4 points]

Solution:

$$\int_1^7 \frac{x^2}{c}\,dx = 1$$
$$\left.\frac{x^3}{3c}\right|_1^7 = 1$$
$$343 - 1 = 3c$$
$$c = \frac{342}{3}$$
$$c = 114$$

(b) Let X be a random variable with PDF

$$f_X(x) = \begin{cases} x/16 & 2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the CDF $F_X(x)$. Be sure to include all cases. [4 points]

Solution:

We have $F_X(x) = \int_{-\infty}^{x} f_X(k)\,dk$. There are 3 cases:

(a) For $x < 2$, $F_X(x) = 0$

(b) For $2 \le x \le 6$,

$$F_X(x) = \int_2^x \frac{k}{16}dk = \frac{1}{16} \cdot \left.\frac{k^2}{2}\right|_2^x = \frac{x^2 - 4}{32}$$

(c) For $x > 6$, $F_X(x) = 1$

Putting them together, we have:

$$F_X(x) = \begin{cases} 0 & x < 2 \\ \frac{x^2-4}{32} & 2 \le x \le 6 \\ 1 & 6 < x \end{cases}$$

(c) As in the previous part, let

$$f_X(x) = \begin{cases} x/16 & 2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\mathbb{E}[4X^2]$.

Again, an answer is considered fully simplified if someone with a calculator but no knowledge of calculus could get an exact number. [4 points]

Solution:

$$\mathbb{E}\left[4X^2\right] = \int_1^3 4x^3/16 = \frac{4x^4}{64}\bigg|_2^6 = \frac{4}{64}(6^4 - 2^4) = \frac{6^4 - 2^4}{16}$$

(d) You are throwing a dart at a circle dartboard with radius 5. You are very good, so your dart always lands on the board. Assume that it is equally likely for a dart to land anywhere on the board. Let X be the shortest distance from the dart to the edge of the board.

Compute the CDF of X, $F_X(x)$. Be sure to include all cases of the CDF. [5 points]

Hint: The shortest distance from the dart to the edge is the radius − the distance to the middle.

Solution:

The CDF will give you the fraction of the circle in the "ring" defined by cutting out the circle of radius $5 - x$ from the center. A bit of geometry will give

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - \frac{(5-x)^2}{25} & 0 \le x \le 5 \\ 1 & 5 < x \end{cases}$$

(e) Let X be a normal random variable with mean 120 and variance 16, and let Y be an normal random variable with mean 90 and variance 9. Assume X and Y are independent. Let $W = X - Y$ be the difference.

Find $\mathbb{E}[W]$ and $\text{Var}(W)$, and list them in the blanks below.

Then, in the box, write $\mathbb{P}(W \ge 18)$ in terms of $\Phi(\cdot)$, the CDF of a $\mathcal{N}(0, 1)$ random variable. [8 points]

Solution:

By linearity of expectation:

$$\begin{aligned} \mathbb{E}[Z] &= \mathbb{E}[X - Y] \\ &= \mathbb{E}[X] - EY \\ &= 120 - 90 \\ &= 30 \end{aligned}$$

Then for variance

$$\begin{aligned} Var(W) &= Var(X) + Var(-Y) \\ &= Var(X) + Var(Y) \\ &= 16 + 9 \\ &= 25 \end{aligned}$$

Then, by closure of normal under addition, $W \sim \mathcal{N}(30, 25)$

The probability that we want to find is $\mathbb{P}(W \geq 18) = \mathbb{P}(Z \geq \frac{-12}{5}) = 1 - \mathbb{P}(Z \leq \frac{-12}{5})$, where $Z = \frac{W-30}{5}$ is a standard normal random variable.
So the answer is $1 - \Phi(-2.4)$