CSE 312 : Autumn 2025 Quiz 3 Form 1 Solutions

Name:

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Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet.
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	Max points
Short Answer	22
Multiple Choice	3
Total	**25**

There are no problems on this page, go to the next one.

1. Short Answer Questions

(a) Suppose I have a random variable X whose PDF is

$$f_X(x) = \begin{cases} ce^{cx} & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the value of c that makes this a valid PDF. [4 points]

Solution:

We need that

$$\int_0^2 ce^{cx} = 1$$

so

$$e^{cx}\big|_0^2 = e^{2c} - 1 = 1$$

Solving, we get that $c = \frac{\ln(2)}{2}$.

(b) Let X be a random variable with PDF

$$f_X(x) = \begin{cases} x/4 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Compute the expectation of X. For this part, an answer is considered fully simplified if someone with a calculator but no knowledge of calculus could get an exact number. [4 points]

Solution:

$$\mathbb{E}[X] = \int_1^3 x^2/4 = x^3/12 \big|_1^3 = \frac{1}{12}(3^3 - 1) = 26/12$$

So the expectation is $\frac{1}{12}(3^3 - 1)$.

(c) As in the previous part, let

$$f_X(x) = \begin{cases} x/4 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Compute the variance of X, and leave your answer in terms of b, your answer for the expectation of X from the previous part. Again, an answer is considered fully simplified if someone with a calculator (and knowledge of b) but no knowledge of calculus could get an exact number. [4 points]

Solution:

$$\mathbb{E}[X^2] = \int_1^3 x^3/4 = \frac{x^4}{16}\bigg|_1^3 = \frac{1}{16}(3^4 - 1) = \frac{80}{16}$$

So the variance is $\frac{1}{16}(3^4 - 1) - b^2$.

(d) An ant of negligible size is located on a line (with negligible width) of length 10. The ant is placed uniformly randomly on the line. Let X be the distance of the ant to the nearest endpoint of the line. Compute the CDF of X, $F_X(x)$. Don't forget to include all cases! [4 points]

Solution:

$$F_X(x) = \begin{cases} x/5 & 0 \le x \le 5 \\ 0 & x < 0 \\ 1 & x > 5 \end{cases}$$

(e) Let Y be a normal random variable with mean 10 and variance 16. Write $\mathbb{P}(2 \le Y \le 18)$ in terms of $\Phi(\cdot)$, the CDF of a $\mathcal{N}(0, 1)$ random variable. [6 points]

Solution:

The probability we care about is $\mathbb{P}(-2 \le Z \le 2)$, where $Z = \frac{Y-10}{4}$ is a standard normal.
This can be written as $\Phi(2) - \Phi(-2)$, or equivalently, $1 - 2\Phi(-2)$, or equivalently, $2\Phi(2) - 1$.

2. Multiple Choice

(a) Let $X \sim$ Unif$(1, 10)$ be a continuous RV and $Y \sim$ Unif$(1, 10)$ be a **discrete** RV. Which of the following are (always) true? Mark ALL that apply. [3 points]
Hint: Remember you have a reference sheet!

- ☐ $\mathbb{E}[X] = \mathbb{E}[Y]$
- ☐ $\Omega_X = \Omega_Y$
- ☐ $\mathbb{P}[X < 10] = \mathbb{P}[Y < 10]$
- ☐ $\mathbb{P}[X > 10] = \mathbb{P}[Y > 10]$

Solution:

First and fourth option. $\mathbb{E}[X] = \frac{1+10}{2} = \mathbb{E}[Y]$ and $\mathbb{P}[X > 10] = 0 = \mathbb{P}[Y > 10]$
For the remaining two options, $\Omega_X = [1, 10] \neq \{1, \ldots, 10\} = \Omega_Y$ (i.e. the first is all real numbers, the second is just integers) and $\mathbb{P}[X < 10] = \mathbb{P}[X \le 10] = 1$, but $\mathbb{P}[Y < 10] = \mathbb{P}[Y \le 9] < 1$.