CSE 312 : Autumn 2025 Final Exam, Form A Solutions

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Instructions

- You have 110 minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes on both sides allowed).
- You are also provided a reference sheet with the exam.
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper. If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- **Please put final answers in provided boxes on longer problems.**
- In general, show us the work you used to get to an answer; explanations will help us award partial credit, but we do not expect explanations at the level we usually require on homeworks.

Simplification Expectations

- Since you don't have a calculator for this exam, you do not have to do simplifications that could be done easily with a scientific calculator. For example, the expression below is simplified enough to be a final answer.

$$\frac{\binom{5}{3} \cdot 17^2}{1-p} + 5^3$$

- However, answers which are much more complicated than the expected answer may receive deductions. For example: $\sum_{i=0}^{n} \binom{n}{i}$ or $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$ are **not** simplified sufficiently; $\int_1^3 x^3 dx$ or $(2t^3)\big|_0^x$ are **not** simplified sufficiently.
- Generally derivatives, integrals, summations, or "..." are not sufficiently simple, unless otherwise indicated.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	**Max points**
Small Questions	22
Random Variables	17
Joint Distributions	12
Conditional Probability	10
Concentration Inequalities	16
MLE	10
Multiple Choice	22
Grading Morale	1
Total	**110**

1. Small Questions [22 points]

(a) There are 6 kids and 5 parents going to a play on a school field trip, and they want to take their seats (all in a row) at a baseball game. How many orderings are possible if...

(i) ... no kid can sit next to another kid (note this means they alternate kid, parent, kid, parent, ..., kid)? [2 points]

Solution:

$6! \cdot 5!$

(ii) ... Stevie and Bobby refuse to sit next to each other, but there are no other restrictions? [2 points]

Solution:

$11! - 10! \cdot 2!$

(b) Salt & Straw has 8 different flavors of ice cream available. You decide to make a mega-sampler (30 scoops, where order does not matter and repetitions are allowed), but you want to have have at most 1 scoop of Thanksgiving flavor (you think meat in ice cream is bad, but you are also a completionist). How many ways are there to make a mega-sampler according to your requirements? [4 points]

Solution:

Stars and Bars. Total number of ways is $\binom{30+8-1}{8-1}$.
The total number of ways where we have at least two of the thanksgiving flavor is $\binom{(30-2)+(8-1)}{8-1}$
So our final answer is the difference of these two quantities:

$$\binom{37}{30} - \binom{35}{28}$$

(c) Let W be a $\mathcal{N}(-10, 4)$ random variable. For what value of c does $\mathbb{P}[-16 < W < -4] = 2\Phi(c)$? [3 points]

Solution:

$Z = \frac{W-(-10)}{\sqrt{4}}$ is a standard normal RV, so the probability we are looking for is the probability

$$\Pr[-3 < Z < 3] = \Phi(3) - \Phi(-3) = \Phi(3) - (1 - \Phi(3)) = 2\Phi(3).$$

(d) Let $X_1, \ldots, X_n$ be continuous uniform random variables taking values in the range $[0, 20]$. Let $Z = \min\{X_1, \ldots, X_n\}$. Compute $\mathbb{P}[Z > z]$ for a fixed $0 < z < 20$. [3 points]

Solution:

$\Pr[Z > z] = (1 - z/20)^n$

(e) Suppose you are typing on a keyboard containing 14 distinct characters, one of which is 'x'. How many 4 letter strings can you type that contain at least two 'x' characters **in a row**? [3 points]
Hint: We know of at least two ways to do this problem: Inclusion/Exclusion or complementary counting.

Solution:

$3 \cdot 14^2 - 2 \cdot 14$ using PIE, or $14^4 - 13^4 - 4(13)^3 - 3(13)^2$. by complementary counting
OR $3 \cdot 13^2 + 4 \cdot 13 + 1$ (3 disjoint cases: 2 x's in a row, 3 x's, 4 x's)

(f) An ant is placed uniformly randomly on a stick of length 8. Let X be the distance to the center of the stick.

(i) On the line segment below, shade in the area(s) at which the ant could appear if $X \leq 1.5$. [1 point]

(ii) Give $F_X(x)$. [4 points] Be sure to include all cases.

Solution:

(a) area should be shaded 1.5 points away from the center on both sides

(b) $F_X(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \leq x \leq 4 \\ 1 & 4 < x \end{cases}$

2. Random Variables [17 points]

You are playing a new video game and want to collect as many gems as possible. The goal of the game is trying to get to 1000 gems by the end of the game. You have the following methods of gem generation at your disposal.

- **Logic Puzzle**: If you solve a logic puzzle, you will be awarded 10 gems (and none if you get it wrong). Your logic skills are rusty, so you have only an 0.8 chance of getting the question right.
- **Coin Flips**: You can instead play a mini-game 3 times. Each of those 3 times, you flip a fair coin. If the coin is heads, you get two gems. However, if it's tails, you lose two gems. This might sounds harsh, but there's a bonus in this game! If you get all three heads, you get 24 (additional) gems.
- **Earthquakes**: You can stand by a fault line for one time unit, which awards you Poi(15) gems.

Let L, C, E be random variables representing the number of gems awarded each of these ways.

(a) You have taken CSE 312, so you wish to find the best method of gem generation. What is the expected number of gem you get from each method? [6 points]

Solution:

(i) L: We have $L = 10X$, where $X \sim$ Ber(0.8). Therefore, we have $\mathbb{E}[L] = 10 \cdot \mathbb{E}[X] = 10 \cdot 0.8 = 8$

(ii) C:

$$\begin{aligned}\mathbb{E}[C] &= \frac{1}{8} \cdot (-6) + \frac{3}{8} \cdot (-2) + \frac{3}{8} \cdot (2) + \frac{1}{8} \cdot (6 + 24) \\ &= 3\end{aligned}$$

(iii) E: $E \sim$ Poi(15). $\mathbb{E}[E] = 15$

(b) Suppose you have 500 gems at the moment and you really liked **Coin Flips**, so after this you will only play **Coin Flips**. What is the least number of rounds r you need to play such that your expected (total) number of gems you have is greater than or equal to 1000?
Write your answer in terms of μ_c, your answer for $\mathbb{E}[C]$ from the previous part. [3 points]

Solution:

$$\begin{aligned}\mathbb{E}\left[500 + \sum_{i=1}^{r} C_i\right] &\geq 1000 \\ 500 + \mathbb{E}\left[\sum_{i=1}^{r} C_i\right] &\geq 1000 \\ \mathbb{E}\left[\sum_{i=1}^{r} C_i\right] &\geq 500 \\ r \cdot \mathbb{E}[C_i] &\geq 500 \\ r &\geq \frac{500}{\mathbb{E}[C_i]} \\ r &\geq \frac{500}{\mu_c}\end{aligned}$$

(c) You realize you'll need to get a bunch of experience points, and so will need to play these many times. You are therefore interested in the variances of each approach. Compute the variances of L and E. [4 points]

Solution:

$\text{Var}(L) = \text{Var}(10X) = 10^2\text{Var}(X) = 10^2 \cdot 0.8 \cdot (1 - 0.8)$
$\text{Var}(E) = 15$

(d) We now analyze the variance of C. Let random variable F be the number of gems won (or lost) from just the flips, and let B be the number of gems won from the bonus (or 0 if you get no bonus). For example, if you get three tails: $F = -6, B = 0$; if you get three heads: $F = 6, B = 24$.

In the lines below, write the variances of F and B. [2 points]

Solution:

$\text{Var}(F) = 4^2 \cdot 3 \cdot \frac{1}{2}^2$
$\text{Var}(B) = 24^2 \cdot \frac{1}{8} \cdot (1 - \frac{1}{8})$
One can see that $F = \sum_{i=1}^{3} F_i$. Where F_i represents number of gems you earn from flip i^{th}. Let $X \sim \text{Ber}(0.5)$. We can see that $F_i = 4X - 2$, so $\text{Var}(F_i) = 4^2 \cdot \text{Var}(X) = 4^2 \cdot \left(\frac{1}{2}\right)^2$. We can assume the coin flips are independent, and so $\text{Var}(F) = 3 \cdot \text{Var}(F_i) = 4^2 \cdot 3 \cdot \left(\frac{1}{2}\right)^2$.

Alternatively, we can write out the PMF of F_i:

$$P(F_i = f) = \begin{cases} 0.5 & \text{if } f = +2 \\ 0.5 & \text{if } f = -2 \end{cases}$$

Then,

$$\begin{aligned} \text{Var}(F_i) &= \mathbb{E}\left[F_i^2\right] - \mathbb{E}\left[F_i\right]^2 \\ &= (4 \cdot 0.5 + 4 \cdot 0.5) - [(2 \cdot 0.5) + (-2 \cdot 0.5)]^2 \\ &= 4 \end{aligned}$$

And again $\text{Var}(F) = 3 * \text{Var}(F_i) = 12$.

For B, we can see that $B = 24X$, where $X \sim \text{Ber}\left(\frac{1}{8}\right)$.

(e) Which of the following best describes $\text{Var}(C)$ in terms of $\text{Var}(B)$ and $\text{Var}(F)$? [2 points]

- ◯ $\text{Var}(C) = \text{Var}(B) + \text{Var}(F)$
- ◯ $(\text{Var}(C))^2 = (\text{Var}(B))^2 + (\text{Var}(F))^2$
- ◯ $\text{Var}(C)$ is neither of the formulas above; it is greater than $\text{Var}(B) + \text{Var}(F)$
- ◯ $\text{Var}(C)$ is neither of the formulas above; it is less than $\text{Var}(B) + \text{Var}(F)$

Solution:

option 3, B and F have positive covariance (you could perform a full computation, but intuitively: when B is above its expectation, you got the bonus, and therefore F must have been above its expectation as well), so if you evaluate $\text{Var}(C) = \text{Var}(B) + \text{Var}(F) + 2\,\text{Cov}(B, F)$, the final term is positive, so $\text{Var}(C)$ is greater.

3. Joint Distributions [12 points]

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{5} & 0 \le x, y \le 2 \\ \frac{1}{5} & -1 \le x, y \le 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute $f_X(x)$. Be sure to include all cases. [5 points]

Solution:

There are 2 cases to consider here:

(i) $-1 \le x \le 0$

$$\begin{aligned} f_X(x) &= \int_{-1}^{0} \frac{1}{5}\, dy \\ &= \frac{y}{5}\Big|_{-1}^{0} \\ &= \frac{1}{5} \end{aligned}$$

(ii) $0 \le x \le 2$:

$$\begin{aligned} f_X(x) &= \int_{0}^{2} \frac{1}{5}\, dy \\ &= \frac{y}{5}\Big|_{0}^{2} \\ &= \frac{2}{5} \end{aligned}$$

Putting everything together, we have:

$$f_X(x) = \begin{cases} \frac{1}{5} & -1 \le x \le 0 \\ \frac{2}{5} & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(b) Compute $\mathbb{E}\left[Y|(X = 1.374)\right]$. [3 points]

Solution:

$$\mathbb{E}\left[Y|X = x\right] = \begin{cases} -\frac{1}{2} & -1 \le x \le 0 \\ 1 & 0 \le x \le 2 \end{cases}$$

This can be seen by symmetry – for any fixed positive x between 0 and 2, the conditional PMF for Y given this fixed value of x is uniform between 0 and 2, so the answer is 1.
Or, you could do it the hard way:

$$\begin{aligned} \mathbb{E}\left[Y|X = 1.374\right] &= \int_0^2 y f_{Y,X}(y, 1.374)/f_X(1.374) dy \\ &= \int_0^2 y(1/5)/(2/5) dy \\ &= y^2/4\big|_0^2 = 1. \end{aligned}$$

(c) Write down an integral expression for $\mathbb{P}(X + Y < 0)$. You should fill in the bounds and function we are integrating, but you do not need to evaluate the integral. [4 points]
Hint: You only need to think about one of the two main cases; this problem is much easier if you draw a doodle of the event.

Solution:

$\mathbb{P}(X+Y<0) = \int_{-1}^{0}\int_{-1}^{0} \frac{1}{5}\, dy\, dx$
Note that when $X+Y<0$, we have $Y<-X$, that is, you are below the line $Y=-X$. The line $Y=-X$ includes all of the second case, but none of the first case.

4. Select All Images With Bots [10 points]

(a) You are working on a CAPTCHA system[1] to detect whether a request came from a human or a bot.
Let T be the event that the request passes your CAPTCHA test, and let B be the event that the request came from a bot. From testing, you have the following probabilities:

- If the request came from a bot, the probability that it passes the test is 0.2.
- If the request came from a human, the probability that they pass the test is 0.9.
- The probability of any given user being a bot is 0.25.

Find the probability a request came from a bot, given that it has passed the test. [4 points]

Solution:

$$\begin{aligned}\mathbb{P}[B \mid T] &= \frac{\mathbb{P}[T \mid B]\,\mathbb{P}[B]}{\mathbb{P}[T]} && \text{Bayes' theorem} \\ &= \frac{\mathbb{P}[T \mid B]\,\mathbb{P}[B]}{\mathbb{P}[T \mid B]\,\mathbb{P}[B] + \mathbb{P}[T \mid B^C]\,\mathbb{P}[B^C]} && \text{LTP} \\ &= \frac{(0.2)(0.25)}{(0.2)(0.25) + (0.9)(1-0.25)}\end{aligned}$$

(b) You've started up your CAPTCHA system on your website, but some bots are still getting through and leaving comments – exactly 100 bot-comments appear each day.

On days where you are busy with other tasks, you briefly skim the comments and delete each bot-comment independently with probability 0.5. On a slow day, you can check much more carefully, and delete each bot-comment independently with probability 0.8. Each day is busy with probability 0.6, and slow otherwise.

What is the expected number of **undeleted** comments at the end of the day? [4 points]

Solution:

Let B be the event that you are busy today, where $\mathbb{P}[B] = 0.6$, $\mathbb{P}[B^C] = 1 - 0.6 = 0.4$ and B, B^C partition the sample space. Let random variable X be the number of **undeleted** comments at the end of the day.

- Given B, X is binomially distributed with parameters $n = 100$ (number of comments) and $p = (1-0.5) = 0.5$ (the probability that each comment is **not** deleted). So $\mathbb{E}[X \mid B] = (100)(0.5)$.
- Given B^C, X is binomially distributed with parameters $n = 100$ and $p = (1-0.8) = 0.2$. So $\mathbb{E}[X \mid B^C] = (100)(0.2)$.

By LTE,

$$\begin{aligned}\mathbb{E}[X] &= \mathbb{E}[X \mid B]\,\mathbb{P}[B] + \mathbb{E}[X \mid B^C]\,\mathbb{P}[B^C] \\ &= (100)(0.5)(0.6) + (100)(0.2)(0.4)\end{aligned}$$

(c) Suppose that you attempt the CAPTCHA test, where each attempt takes you Exponential$(1/30)$ seconds. Furthermore, the first time you attempt a CAPTCHA, it will take you 20 additional seconds to read the directions (and after the first time, you ignore the directions). What is the expected amount of time that it takes for you to attempt 10 CAPTCHAs, including the initial time spent on reading the directions once? [2 points]

Solution:

Let Y_i be the number of seconds spent on the ith CAPTCHA, $i \in \{1, \ldots, 10\}$. Each Y_i has expectation 30 by definition.
Let $Y = 20 + \sum_i Y_i$ be the total time (in seconds) that you spend on attempting 10 CAPTCHAs, including

[1] CAPTCHAs are things like "pick which pictures have a traffic light" tests to tell whether you're a human.

the 20 seconds spent on directions. By linearity of expectation,

$$\begin{aligned}\mathbb{E}[Y] &= \mathbb{E}\left[20 + \sum_{i=1}^{10} Y_i\right] = 20 + \sum_{i=1}^{10} \mathbb{E}[Y_i] \\ &= 20 + 10(30)\end{aligned}$$

5. Knitting Requires Concentration, Again [16 points]

It's winter, and Robbie is working steadily on knitting a scarf. On day i, he knits R_i rows of the scarf, where R_i is a random variable with expectation 5 and variance 3. Treat the R_i variables as continuous (i.e., you could make a fractional part of a row).

(a) Use Markov's inequality to bound the probability that $R_i < 7.5$. [4 points]

Solution:

$\mathbb{P}[R_i < 7.5] = 1 - \mathbb{P}[R_i \geq 7.5]$. By Markov,

$$\mathbb{P}[R_i \geq 7.5] \leq \frac{\mathbb{E}[R_i]}{7.5} = \frac{5}{7.5}$$
$$\mathbb{P}[R_i < 7.5] \geq 1 - \frac{5}{7.5}$$

(b) Use Chebyshev's inequality to bound the probability that $R_i < 2$ or $R_i > 8$. [4 points]

Solution:

If $R_i < 2$ or $R_i > 8$, then R_i is more than 3 away from its mean of 5. So

$$\mathbb{P}[R_i < 2 \text{ or } R_i > 8] = \mathbb{P}[|R_i - \mathbb{E}[R_i]| \geq 3]$$
$$\leq \frac{\text{Var}(R_i)}{3^2} = \frac{3}{3^2}$$

(c) Robbie's getting much closer to completing the scarf. He decides to focus: now each day he knits with probability 0.6 (independently of other days). If he knits on a given day, he'll complete exactly 10 lines of knitting. If he doesn't knit that day, he knits zero lines.

Over the next 30 days, he hopes to knit at least 100 lines. Let X be the total number of lines he knits. In the following parts, we will use Chernoff to bound the probability that he knits strictly less than 100 lines.

(i) In English, detail the random variable that you will bound using Chernoff, i.e. give it a name (like X, Y, Z, etc.) and describe what it represents in terms of the problem statement. [1 point]

(ii) Calculate the expectation of your random variable in (i). [2 points]

(iii) Finally, bound the probability that Robbie knits less than 100 lines. [5 points]

Solution:

Let $Y_i \sim \text{Ber}(0.6)$ indicate whether Robbie knits on the ith day, for $i \in \{1, \ldots, 30\}$. Then $Y = \sum_i Y_i \sim \text{Bin}(30, 0.6)$ equals the total number of days where he knits, and $X = 10Y$.

We want to bound $\mathbb{P}[X < 100] = \mathbb{P}[X \leq 99] = \mathbb{P}[10Y \leq 99] = \mathbb{P}[Y \leq 9.9]$, where Y is the RV that can be bounded using Chernoff (a sum of independent Bernoulli RVs). Note that X can only increase in increments of 10, so it is discrete.

Note that $\mathbb{P}[Y \leq 9.9] = \mathbb{P}[Y \leq 9]$, as Y is only supported on integers. $\mu = \mathbb{E}[Y] = (30)(0.6)$, so the δ needed for Chernoff (left tail) is

$$9 = (1-\delta)\mu \implies 1 - \frac{9}{30(0.6)} = \delta \quad 0 \leq \delta \leq 1$$

and

$$
\begin{aligned}
\mathbb{P}[X < 100] = \mathbb{P}[Y \leq 9] &= \mathbb{P}[Y \leq (1-\delta)\mu] \\
&\leq \exp\left(-\frac{\delta^2 \mu}{2}\right) \\
&= \exp\left(-\frac{\left(1 - \frac{9}{30(0.6)}\right)^2 (30)(0.6)}{2}\right)
\end{aligned}
$$

6. Maximum Knitting [10 points]

Alice and Bob start a knitting business. They each are capable knitters, but they have to knit quickly to meet demand, which means they sometimes make mistakes. Suppose that Alice makes $\text{Poi}(3)$ mistakes per project, while Bob makes $\text{Poi}(2)$ mistakes per project.

They each have a pile of completed projects, but you aren't sure whose pile is whose. You decide to use maximum likelihood estimation to determine whether Alice or Bob is the creator of the pile right in-front of you (i.e., $\theta \in \{\text{Alice}, \text{Bob}\}$ is the true parameter, and you will use MLE to find $\hat{\theta}$, a prediction of θ).

(a) You observe that the first pile has 5 projects, where each project has 2, 3, 2, 1, and 3 errors respectively. Write the likelihood that this is Alice's pile. [4 points]

Solution:

Since $\lambda = 3$ for Alice, let $p(k) = e^{-3}\frac{3^k}{k!}$, where the likelihood that this pile is Alice's is

$$\mathcal{L}(\text{observed pile}; \text{Alice}) = p(2)^2 \cdot p(3)^2 \cdot p(1)$$

(b) Suppose that you picked up 5 projects and all had 3 or more errors. Which of the following is the best description of the MLE for this sample? [2 points]

◯ $\hat{\theta} = \text{Alice}$
◯ $\hat{\theta} = \text{Bob}$
◯ $\hat{\theta}$ is more likely to be Alice than Bob, but we can't be sure which it is.
◯ $\hat{\theta}$ is more likely to be Bob than Alice, but we can't be sure which it is.

Solution:

1st option. The pile being Alice's will maximize the likelihood that we saw those errors, since for a single project, Alice has a higher probability of making 3+ errors.

(c) When finding an MLE, why do we take the log of the likelihood function? Mark ALL that apply [2 points]

☐ It (usually) makes taking derivatives easier while calculating the MLE.
☐ It (usually) makes integrating easier while calculating the MLE.
☐ It is required (i.e., your MLE may be incorrect if you skip the step).

Solution:

1st option only

(d) Suppose you are working with a <u>continuous</u> distribution. When finding an MLE for a relevant parameter, when do we take the natural logarithm ln instead of $\log_2$ or $\log_{10}$? [2 points]

◯ You must always use ln (not $\log_2$ or $\log_{10}$), or you will get the wrong MLE.
◯ You cannot use ln (you must use $\log_2$ or $\log_{10}$) in order to get the correct MLE.
◯ You cannot take the log (natural or otherwise) at all when finding an MLE.
◯ Any log base can be used; we pick whichever is most convenient.

Solution:

4th option

7. Multiple Choice [22 points]

For the questions below,

- Questions with ◯ Circles have exactly one best answer. **Fully fill in** the circle for the one best answer.
- Questions with ☐ squares are "mark all that apply" questions. **Fully fill in** the square for all correct options.

(a) Suppose I roll a fair 6-sided die, and then my friend rolls a fair k-sided die, where k was the outcome of my die roll. Let X be the outcome of my die roll, and Y be the outcome of my friend's die roll. Select ALL that apply. [3 points]

☐ Y is a discrete uniform random variable
☐ $p_{X,Y}(x,y) = \frac{1}{6x}$ for all integers x, y such that $0 \leq y \leq x \leq 6$
☐ $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$
☐ $X|(Y = 3)$ is a uniform random variable
☐ $Y|(X = 3)$ is a uniform random variable

Solution:

2,5

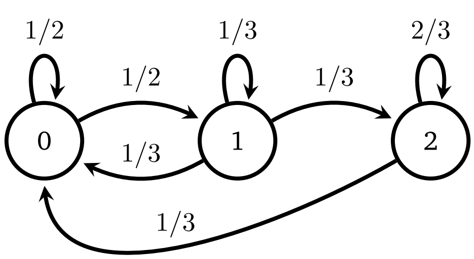
(b) Let X be a random variable. Then $\mathbb{E}\left[(X-4)^2\right] \geq (\mathbb{E}[X-4])^2$. [3 points]

◯ Always
◯ Sometimes
◯ Never

Solution:

Always. If you subtract the RHS, you have the variance of $X - 4$. The variance of a random variable is always nonnegative.

(c) Consider the following Markov Chain:



What is the probability we are in state 0 at timestep two, if we were in state 1 at timestep zero? [3 points]

◯ $(1/3)^2 + (1/3)(1/2) + (1/2)^2$
◯ $(1/3)^2 + (1/3)(1/2)$

- ◯ $2(1/3)^2 + (1/3)(1/2)$
- ◯ $2(1/3)^2 + (1/2)^2$

Solution:

option 3.

(d) Using a trigram model trained on the following sentence, what is the probability of $\mathbb{P}$ [probability|I love]? [3 points]

"START START I love probability almost as much as I love TikTok and I love probability more than I love Roblox STOP"

- ◯ 1/2
- ◯ 1/4
- ◯ 1/10
- ◯ 1/11

Solution:

1/2

(e) You repeatedly roll a fair 20-sided die until you roll the first 20. Let t be the probability that you see your first 20 on the 8th roll.
Suppose that your first two rolls are 1 and 2 respectively. Knowing this, what is the probability that you see your first 20 on the 10th roll? [3 points]

- ◯ t^2
- ◯ t
- ◯ $t^2(1-t)$
- ◯ $t(1-t)^9$
- ◯ The events described are disjoint, so you can't express one in terms of the other.

Solution:

t, since geometric processes are memoryless. In other words, let X (geo) be the number of trials (rolls) until success (20); if we know that our first two trials were failures, $X > 2$, the probability of succeeding on the 10th trial is equivalent to the probability of succeeding after 8 more trials, which is exactly t.

(f) Suppose you have a binomial random variable $X \sim \text{Bin}(1000, 0.3)$; you wish to estimate $\mathbb{P}[X \geq 250]$ using the Central Limit Theorem. Which of the following is the event you should use to apply a continuity correction? [3 points]

- ◯ $\mathbb{P}[X \geq 250.5]$
- ◯ $\mathbb{P}[X \geq 249.5]$
- ◯ You should not apply a continuity correction in this case.
- ◯ The Central Limit Theorem cannot be used here as 250 is less than $\mathbb{E}[X]$.

Solution:

$X \geq 249.5$

(g) For any continuous random variable X and any constant $k > 0$ we have that $\mathbb{P}[X \leq k] = \mathbb{P}[X^2 \leq k^2]$. [2 points]

◯ True
◯ False

Solution:

False

(h) For $X \sim \mathcal{N}(2, 100)$, we have that $\mathbb{P}[X \geq 10] \leq 1/5$ by Markov's inequality. [2 points]

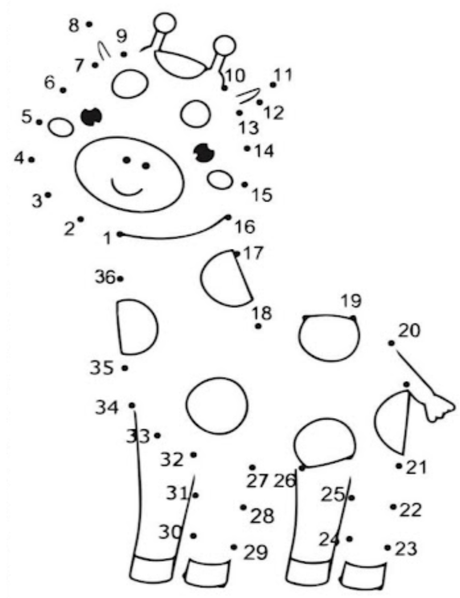
◯ True
◯ False

Solution:

False. Markov's requires the RV to be nonnegative.

8. Grading Morale [1 point]

Put something on this page, finish the image below, write a message to the teaching staff below, or do something else. Depending on your mood, this might be a poem, the statement of your favorite theorem, the statement of your least favorite theorem, or a piece of art. As long as you make some mark on this page, you will get the point. Looking at these helps keep the TAs happy while grading.



Have a relaxing winter break!

Use this page for extra space if needed. Be sure to tell us to look here on the original problem.