CSE 312 – Section 1

Summer 2026

Review

a) **Sum rule.** If you can choose from EITHER one of n options, OR one of m options with NO overlap with the previous n, then the number of possible outcomes of the experiment is $n + m$.

b) **Product rule.** In a sequential process with m steps, if there are n_1 choices for the 1st step, n_2 choices for the 2nd step (given the first choice), ..., and n_m choices for the mth step (given the previous choices), then the total number of outcomes is $n_1 n_2 ... n_m$.

c) **Permutations.** The number of ways to order n distinct elements is $n!$.

d) k**-permutations.** The number of ways to choose a sequence of k distinct elements from a set of n elements is $P(n, k) = \frac{n!}{(n-k)!}$. (Note that order matters.)

e) **Subsets.** The number of ways to choose a k-element subset of a set of n elements (where order does not matter) is $C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$.

f) **Complementary counting.** If S is a set and is the disjoint union of two sets A and B, then you can find the size of A by finding the size of S and then subtracting the size of B.

g) **Binomial theorem.** $\forall x, y \in \mathbb{R}, \forall n \in \mathbb{N}$: $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

h) **Inclusion-exclusion.**

$$\begin{aligned} |A \cup B| &= |A| + |B| \\ &\quad - |A \cap B|. \end{aligned}$$

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| \\ &\quad - |A \cap B| - |A \cap C| - |B \cap C| \\ &\quad + |A \cap B \cap C| \end{aligned}$$

$$\begin{aligned} |A \cup B \cup C \cup D| &= |A| + |B| + |C| + |D| \\ &\quad - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\ &\quad + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\ &\quad - |A \cap B \cap C \cap D| \end{aligned}$$

and so on!

i) **Multinomial coefficients.** Suppose there are n objects, but only k are distinct, with $k \leq n$. (For example, "godoggy" has $n = 7$ objects (characters) but only $k = 4$ are distinct:

(g, o, d, y)). Let n_i be the number of times object i appears, for $i \in \{1, 2, \ldots, k\}$. (For example, $(3, 2, 1, 1)$, continuing the "godoggy" example.) The number of distinct ways to arrange the n objects is

$$\binom{n}{n_1, \ldots, n_k} = \frac{n!}{n_1! \cdot \ldots \cdot n_k!}.$$

Plan for section

Administrivia + Introductions

- Icebreaker
- Section: Solutions to all the problems on this worksheet will be available on Thursday evening.
- Office hours: Times and locations on website, start Friday, June 26.
- Pset 1 is out and due next Wednesday (July 1). Submit all parts on Gradescope.
- Homework: Use of LaTeX highly encouraged. See website for more information on formatting your solutions to problem sets.
- Read syllabus for late day policy.
- Introduction to your TAs!

Questions about material covered so far?

Problems to be covered in section

We may not get through all of these.

- Problem 1 (Seating)
- Problem 4 (Escape the Professors)
- Problem 3 (HBCDEFA)
- Problem 15 (Binomial Theorem)
- Problem 10 (Extended Family Portrait)

Be sure to check out all the remaining problems before you do your homework.

1 Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if ...

a) ...all couples are to get adjacent seats?

b) ...anyone can sit anywhere, except that one couple insists on *not* sitting in adjacent seats?

2 Weird Card Game

In how many ways can a pack of 52 cards be dealt to 13 players, 4 to each, so that every player has 1 card of each suit?

3 HBCDEFGA

How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

4 Escape the Professor

There are 6 security professors and 7 theory professors taking part in an escape room. If 4 security professors and 4 theory professors are chosen and paired off, how many pairings are possible?

5 Birthday Cake

A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

6 Full Class

There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

7 Paired Finals

Suppose you are to take a CSE 312 final in pairs. There are 100 students in the class and 8 TAs, so 8 lucky students will get to pair up with a TA. Each TA must take the exam with some student, but two TAs cannot take the exam together. How many ways can they pair up?

8 Photographs

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

9 Rabbits!

Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

10 Extended Family Portrait

A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?

11 Subsubset

Let $[n] = \{1, 2, ..., n\}$ denote the first n natural numbers. How many (ordered) pairs of subsets (A, B) are there such that $A \subseteq B \subseteq [n]$?

12 Divide Me

How many numbers in [360] are divisible by:

a) 4, 6, and 9?

b) 4, 6, or 9?

c) Neither 4, 6, nor 9?

13 Practice with Sets

a) For each one of the following sets, give its **cardinality**, i.e., indicate how many elements it contains:

- $A = \emptyset$
- $B = \{\emptyset\}$
- $C = \{\{\emptyset\}\}$
- $D = \{\emptyset, \{\emptyset\}\}$

b) Let $S = \{a, b, c\}$ and $T = \{c, d\}$. Compute:

- $S \cup T$
- $S \cap T$
- $S \setminus T$
- $2^{S \setminus T}$
- $S \times T$

c) Is it always true that $|A \setminus B| = |A| - |B|$?

14 Basic Counting

a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is uniquely determined by the first 15 digits). How many credit-card numbers are there?

b) How many positive divisors does $1440 = 2^5 3^2 5$ have?

c) How many ways are there to arrange the CSE 311 staff on a line (11 TAs, two professors) for a group picture?

d) How many ways are there to arrange the CSE 311 staff on a line so that Professors Tessaro and Beame are at the two ends of the line?

15 Binomial Theorem

What is the coefficient of z^{36} in $(-2x^2yz^3 + 5uv)^{312}$?

16 Multinomial Coefficients

How many ways can we arrange the letters in 'TEDDYBEAR'?