

Welcome to 312! 🥳 You're early!

Want a copy of these slides to take notes?

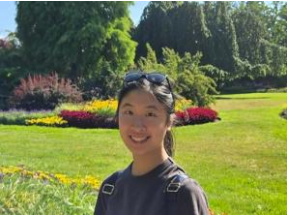
You can download them from the webpage [cs.uw.edu/312](https://cs.uw.edu/312)

Feel free to introduce yourself to the people sitting around you!

# Introduction and Counting

CSE 312 26Su  
Lecture 1

# Staff



Instructor: Jolie Zhou

Graduate student in CS at UW

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Office hours: WF 1:15-2:30pm

## TAs

Escher Crawford  
Darin Ershov

Krishna Khandelwal  
Vlad Murad

# Logistics - Lecture

Lectures are MWF

- 12:00-1:00pm in HRC 155
- Slides will be posted on the website
- Recorded on Panopto

Please participate and ask questions!

# Logistics - Section

Sections on Thursdays starting this week

Both sections are at 12pm (different locations)

Sections will **not** be recorded – we want you to be able to ask questions and give feedback without worrying about being recorded.

Handouts and solutions will be posted.

Bring laptop/phone and pencil/pen to section. (We will bring paper for you to write on.)

# Concept checks

## Concept checks after each lecture

Released by 1:30pm (30 mins after lecture). Must be done before the next lecture.

Simple questions to reinforce concepts taught in each class.

Keep you engaged and on top of the material throughout the week, so that homework becomes less of a hurdle.

Do them as soon as you can after lecture. Really important.

## Important!

If you don't submit by the due date, you won't be able to see the solutions until at least a week later.

Submit, even if you don't answer any of the questions!! You can resubmit as many times as you want before the deadline.

# Exams

6 quizzes in lecture (15 minutes each)

First 15 minutes of Friday lectures, see specific days on course calendar

2 lowest dropped

Quiz 1: this week on **Friday June 26<sup>th</sup>**

Midterm: **Wednesday July 22<sup>nd</sup>**

More resources to be posted later

Final Exam (2 parts): **Thursday August 20<sup>th</sup> , Friday August 21<sup>st</sup>**

More resources to be posted later

# Grading

Concept checks (15%)

7 Homeworks / Psets (20%)

Mostly math. A few will have coding components. Pset 1 out on Wednesday.

6 Quizzes (15%): select Friday lectures

Midterm (20%): Wednesday, July 22, 12-1pm in lecture

Final (30%): Thursday, August 20, 12-1pm in section  
Friday, August 21, 12-1pm in lecture

Small extra credit opportunities

# Syllabus

## Late submission

2 penalty free late days on every homework

Must have meaningful submission by deadline

## Grade guarantees

Must meet minimum exam averages to guarantee grades

See specific breakdown on syllabus

See syllabus for more info on academic integrity, course tools, etc.

Ask questions on Ed!

# Accommodations

If you have something significant happen during the quarter, please reach out to use as soon as possible!

For now:

- Fill out DRS accommodations if needed
- Fill out religious accommodations request form if needed
- Email me if you have other accessibility concerns that are not covered by DRS (!)

What is this class about?



**Foundations of Computing II**

=

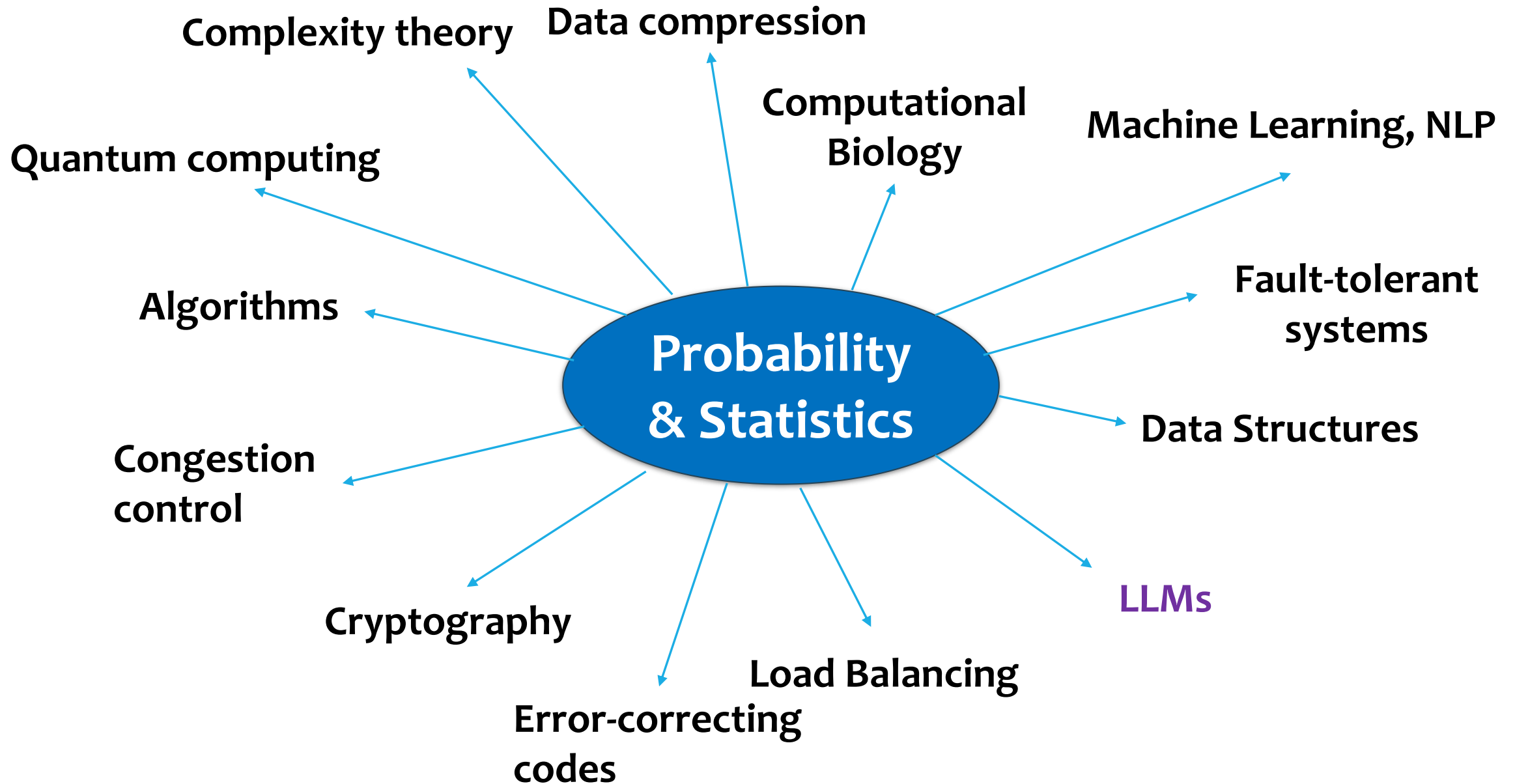
**Introduction to Probability & Statistics**  
for computer scientists

What is probability??

Why probability?!

# Connections

+ much more!



# Beyond important applications in computer science and engineering

Two additional arguments for studying hard and really learning the material in this class:

The economic argument (the “need”)

The “fun/useful in real life” argument (the “want”)

# The economic argument and the elephant in the room

Generative AI (like ChatGPT, Claude or Gemini) can solve 95% of the homework problems in this class in seconds.

Thus, the market value of “getting the answer” is effectively \$0.00

Employers do not pay humans for skills that are free via an API

So why are you here?

# The economic argument and the elephant in the room

Generative AI (like ChatGPT, Claude or Gemini) can solve 95% of the homework problems in this class in seconds.

Thus, the market value of “getting the answer” is effectively \$0.00

Employers do not pay humans for skills that are free via an API

So why are you here?

You are here to learn how to be the **Pilot**, not the **Passenger**

# The Pilot vs Autopilot Analogy

Flying a plane on autopilot is easy. Pilots are paid high salaries not for the 99% of the time the weather is clear, but for the 1% of the time the system fails.

## The gap

AI is excellent at the “average” case. It fails catastrophically at “edge cases” – tail risks and black swan events.

## The skill

Probability is your instrument panel. If you don't know how to read the instruments (e.g., *variance, conditional probability, Bayes theorem*), you cannot take over when the AI enters turbulence (e.g., spot a subtle probabilistic error in outputs, a hallucinated *confidence interval*)

# The Changing Interview Landscape

Interviews are adapting to the post-AI world too.

## The test

An interviewer might not ask you to solve a problem. They might ask you to defend a solution.

Example: *"Why did you model this as a Poisson distribution? What happens to your risk model if the correlation shifts by 0.1?"*

## The result.

An LLM can give you the formula, but it cannot give you the intuition to defend it under pressure.

# For us: AI in education

For all these above reasons, you are NOT allowed to use AI tools on homeworks in this class (ChatGPT, Gemini, Claude, etc.)

At the same time, using AI carefully for learning and as a collaborator can be helpful, like a teaching assistant. It can:

- help explain the material we are covering

- walk through section worksheet solutions

- generate extra practice problems

But it's hard to tell what you may be cognitively offloading

Consider it a tool for conceptual questions and not to automate your workload.

# Summary: why you should take this course seriously

Important applications in computer science and engineering.

Crucial for success in future CSE (and other) courses.

Crucial for success in your future career, aka the economic argument (the "need")

The "fun/useful in real life" argument (the "want")

# Content

## Combinatorics (*fancy* counting)

Permutations, combinations, inclusion-exclusion, pigeonhole principle

## Formal definitions for Probability

Probability space, events, conditional probability, independence, expectation, variance

## Common patterns in probability

Equations and inequalities, “zoo” of common random variables, tail bounds

## Continuous Probability

pdf, cdf, sample distributions, central limit theorem, estimating probabilities

## Applications

Across CS, but with some focus on ML.

# Themes

Precise mathematical communication

Both reading and writing dense statements.

Probability in the “real world”

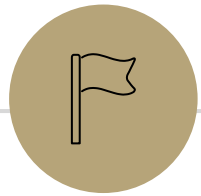
A mix of CS applications

And some actual “real life” ones.

Refine your intuition

Most people have some base level feeling of what the chances of some event are.

We’re going to train you to have better gut feelings.



# Counting



# Why Counting?

We can't always just list out possible options and count it up!

Combinatorics: techniques for counting things that are difficult to count

## Basics of **algorithm analysis**

Is this problem brute-forceable?

Estimation of how many operations an operation like find() would require

## Basics of **probability theory**

Usually, "What are the chances of X" =

how many ways can X occur

how many ways can X occur + how many ways can X not occur

# Remember sets from 311?

A set is an **unordered** list of elements, ignoring repeats.

$\{1,2,3\}$  is a set. It's the same set as  $\{2,1,3\}$ .

$\{1,1,2,3\}$  is a very confusing way of writing the set  $\{1,2,3\}$ .

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The **cardinality** of a set is the number of elements in it.

$\{1,2,3\}$  has cardinality 3

$$|\{1,2,3\}| = 3.$$

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The **cardinality** of a set is the number of elements in it.

$\{1,2,3\}$  has cardinality 3

$$|\{1,2,3\}| = 3.$$

We're going to learn counting techniques to count the size of more complex sets!

# Counting Rule #1 (lunchtime!)

How many options do I have for lunch?

I could go to **Delfino's** where there are **6 pizzas** I choose from, or I could go to **Supreme** where there are **4 pizzas** I choose from (and **none of them are the same between the two**).

How many total choices?

# Counting Rule #1 (lunchtime!)

**What are we counting here?**

We're counting the size/cardinality of the set of all possible lunches

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$$6 + 4 = 10$$

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How many total choices?

$$6 + 4 = 10$$

**Sum Rule:** If you are choosing one thing between  $n$  options in one group and  $m$  in another group with no overlap, the total number of options is:  $n + m$ .

# Counting Rule #2 (coffee)

Now I'm sleepy!

I decide to get coffee. My coffee is always:

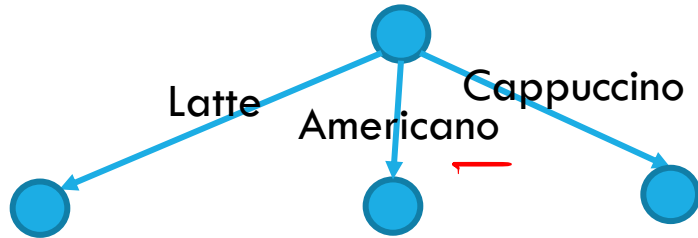
One of three bases (latte, americano, cappuccino).

One of two preparations (iced or hot)

One of four syrups (caramel, peppermint, hazelnut, gingerbread)

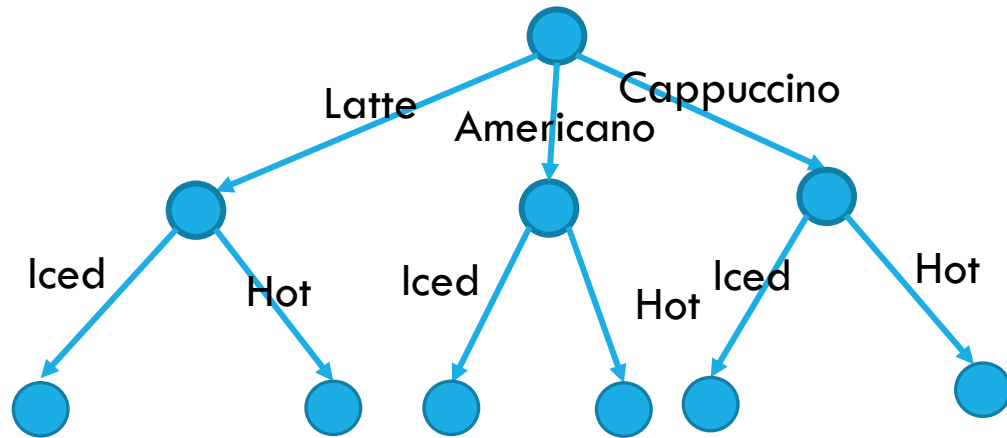
How many possible coffee orders do I have?

# Counting Rule #2 (coffee)



**Step 1:** choose one of the three bases.

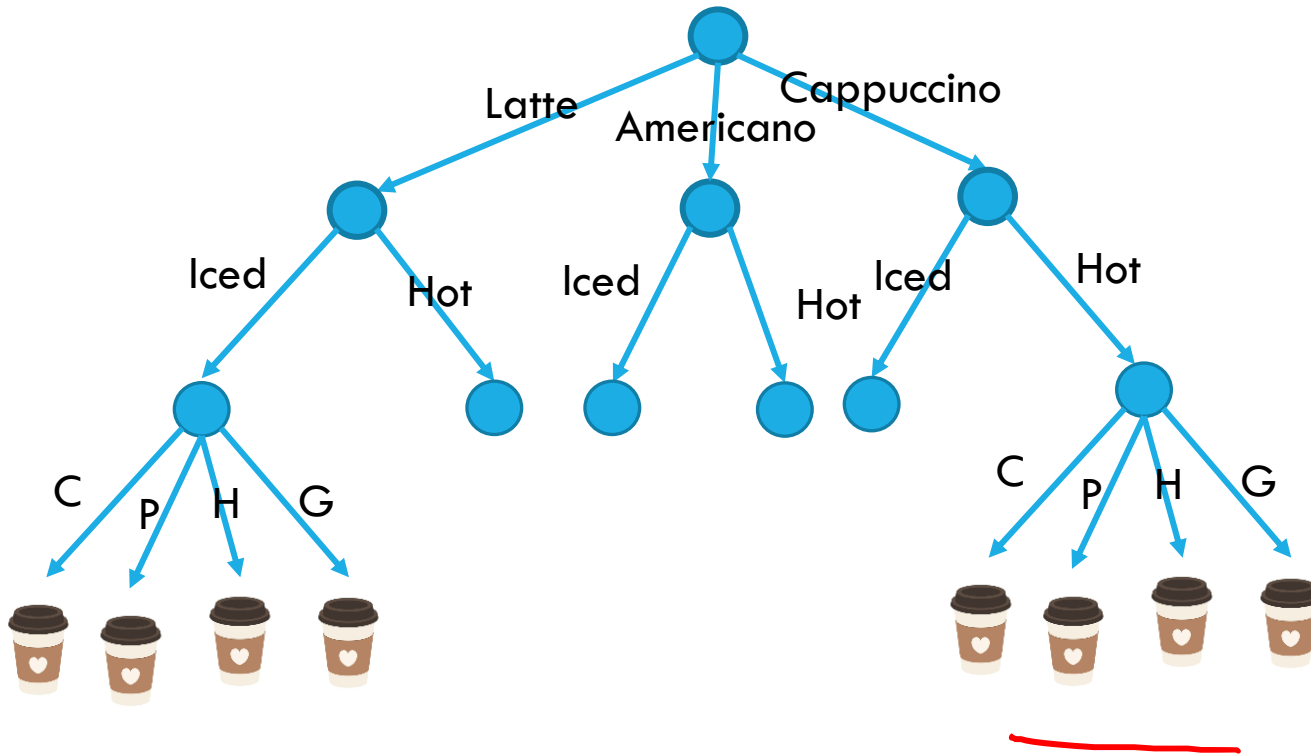
# Counting Rule #2 (coffee)



**Step 1:** choose one of the three bases.

**Step 2:** regardless of step 1, choose one of the two preparations.

# Counting Rule #2 (coffee)

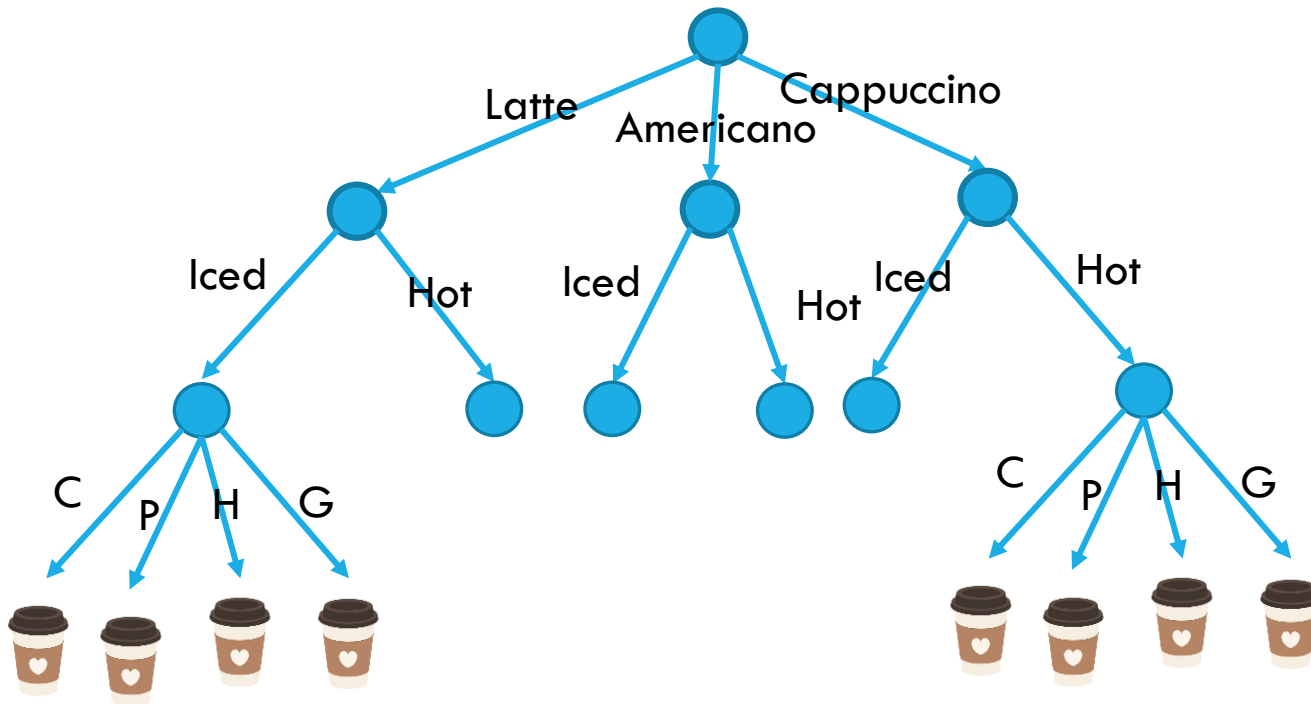


**Step 1:** choose one of the three bases.

**Step 2:** regardless of step 1, choose one of the two preparations.

**Step 3:** regardless of steps 1 and 2, choose one of the four syrups.

# Counting Rule #2 (coffee)



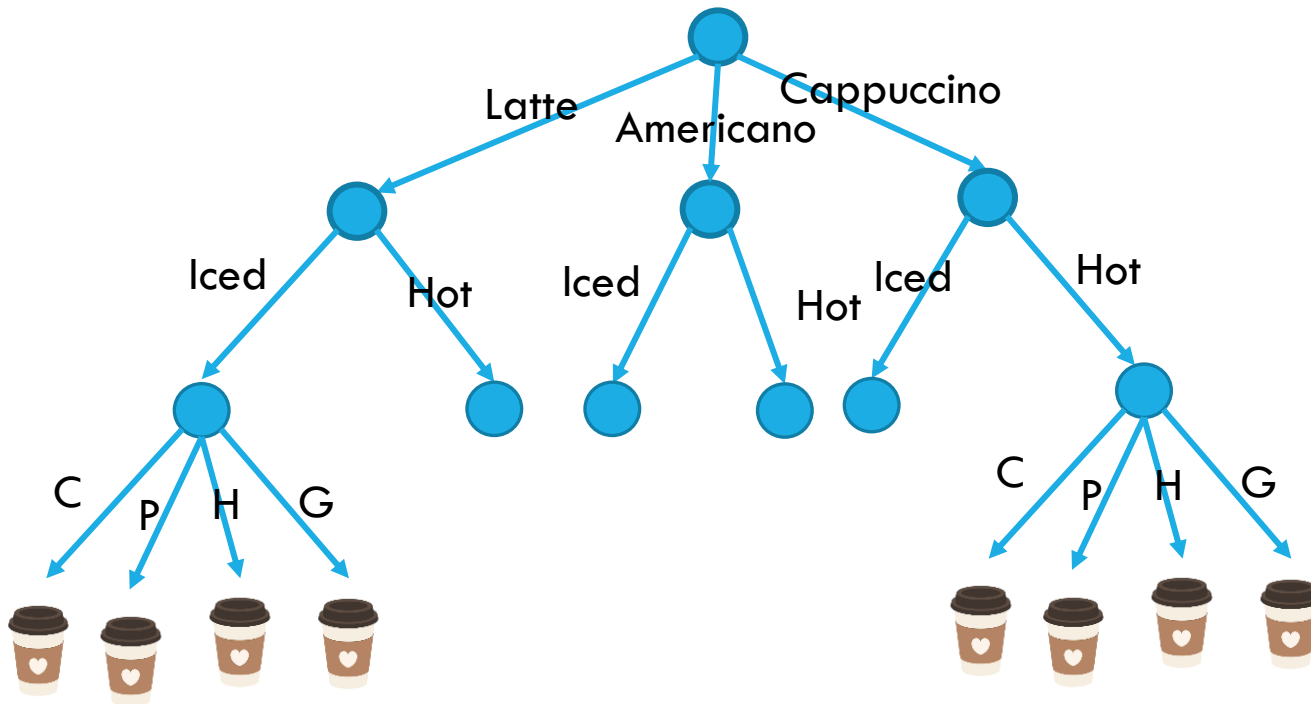
**Step 1:** choose one of the three bases.

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$$3 \cdot 2 \cdot 4 = 24.$$

# Counting Rule #2 (coffee)



**Step 1:** choose one of the three bases.

**Step 2:** regardless of step 1, choose one of the two preparations.

**Step 3:** regardless of steps 1 and 2, choose one of the four syrups.

$$3 \cdot 2 \cdot 4 = 24.$$

**Product Rule:** If you have a sequential process, where step 1 has  $n_1$  options, step 2 has  $n_2$  options, ..., step  $k$  has  $n_k$  options, and you choose one from each step, the total number of possibilities is  $n_1 \cdot n_2 \cdots n_k$

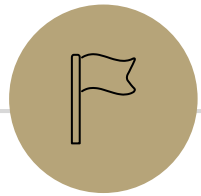
# Counting Rules

Sum Rule: If you are choosing one thing between  $n$  options in one group and  $m$  in another group with no overlap, the total number of options is:  $n + m$ .

★ Used when we're counting something when we have cases that don't overlap ★

Product Rule: If you have a sequential process, where step 1 has  $n_1$  options, step 2 has  $n_2$  options, ..., step  $k$  has  $n_k$  options, and you choose one from each step, the total number of possibilities is  $n_1 \cdot n_2 \cdots n_k$

★ Used when we're counting something with a sequential process ★



# Application of the Product Rule

# Example – Cartesian Products

Remember Cartesian products?

$$S \times T = \{(x, y) : x \in S, y \in T\}$$

$$\{1, 2\} \times \{a, b, c\} = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

# Example – Cartesian Products

Remember Cartesian products?

$$S \times T = \{(x, y) : x \in S, y \in T\}$$

$$\{1, 2\} \times \{a, b, c\} = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

How big is  $S \times T$ ? (i.e. what is  $|S \times T|$ ?)

Step 1: choose the element from  $S$ .

Step 2: choose the element from  $T$ .

Total options:  $|S| \cdot |T|$

# Example – Power Sets

$$\mathcal{P}(S) = \{X: S \subseteq X\}$$

$$\mathcal{P}(\{1,2,3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

How many subsets are there of  $S$ , i.e. what is  $|\mathcal{P}(S)|$ ?

$$\begin{array}{ccccccc} e_1 & , & e_2 & & e_3 & & e_{|S|} \\ \boxed{2} & \times & \boxed{2} & \times & \boxed{2} & \times \dots \times & \boxed{2} = \boxed{2^{|S|}} \end{array}$$

# Example – Power Sets

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$$\mathcal{P}(\{1,2,3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

How many subsets are there of  $S$ , i.e. what is  $|\mathcal{P}(S)|$ ?

If  $S = \{e_1, e_2, \dots, e_{|S|}\}$

Step 1: is  $e_1$  in the subset?

Step 2: is  $e_2$  in the subset?

...

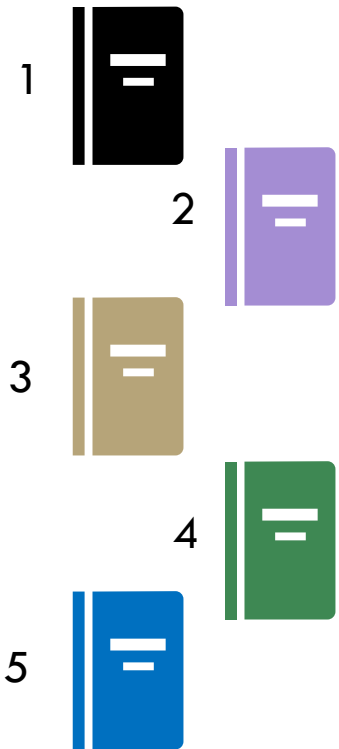
Step  $|S|$ : is  $e_{|S|}$  in the subset?

$2 \cdot 2 \cdots 2$ ,  $|S|$  times, i.e.,  $2^{|S|}$ .

# Example – Assigning Books

We have 5 books to split to 3 people (Alice, Bob, and Charlie)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).



Alice



Bob

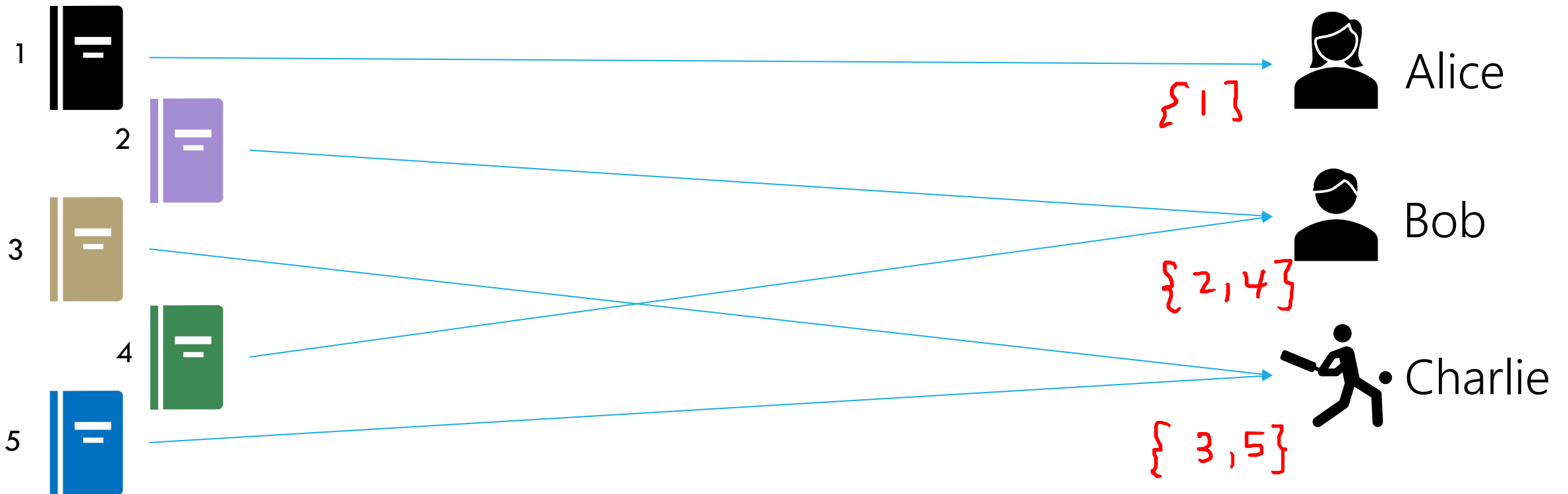


Charlie

# Example – Assigning Books

There are 3 people (Alice, Bob, and Charlie) split up 5 books

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).



# Example – Assigning Books

We have 5 books to split to 3 people (Alice, Bob, and Charlie)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).

Attempt 1: We're choosing subsets!

1. Alice could get any of the  $2^5 = 32$  subsets of the books.
2. Bob could get any of the  $2^5 = 32$  subsets of the books.
3. Charlie could get any of the  $2^5 = 32$  subsets of the books.

Total is product of those three steps  $32 \cdot 32 \cdot 32 = 32768$

# Example – Assigning Books

Attempt 1: We're choosing subsets!

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Charlie could get any of the  $2^5 = 32$  subsets of the books.

Total is product of those three steps  $32 \cdot 32 \cdot 32 = 32768$

We overcounted! **This doesn't account for restriction of 1 book->1 person**

If Alice gets {1,2}, Bob can't get any subset, he can only get a subset of {3,4,5}. And Charlie's subset is just whatever is leftover after Alice and Bob get theirs...

# Example – Fixing All The Books

You could

List out all the options for Alice.  $2^5$

For each of those (separately), list all the possible options for Bob and Charlie.

Use the Sum rule to combine.

case: A  
case B<sub>1</sub>  
case B<sub>2</sub>

~OR~ you could come at the problem from a different angle.

# Example – Fixing All The Books

Instead of figuring out which books Alice gets, choose book by book which person they go to.

Step 1: Book 1 has 3 options (Alice, Bob, or Charlie).

Step 2: Book 2 has 3 options (A, B, or C)

...

Step 5: Book 5 has 3 options.

Total: 3<sup>5</sup>.

# Example – Repeated letters

How many length 5 sequences are there consisting of elements from  $\{A, B, C, D, E\}$  with repeats allowed?

Step 1: 5 options for the *first* character.

Step 2: 5 options for the *second* character.

Step 3: 5 options for the *third* character.

Step 4: 5 options for the *fourth* character.

Step 5: 5 options for the *fifth* character.

AAAAA  
ABCD A

$$5^5$$

# ~~$k$ -permutation~~

Seq -

## $k$ -sequences

$n^k$  length  $k$  sequences from an alphabet of size  $n$ , with  
repeats allowed

# Example – Distinct letters

How many length 5 sequences are there consisting of distinct elements of  $\{A, B, C, D, E\}$ . (in other words, rearranging these 5 elements)

Step 1: 5 options for the *first* character.

Step 2: 4 (remaining) options for the *second* character.

Step 3: 3 (remaining) options for the *third* character.

Step 4: 2 (remaining) options for the *fourth* character.

Step 5: 1 (remaining) options for the *fifth* character.

B D

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Step 4: 2 (remaining) options for the *fourth* character.

Step 5: 1 (remaining) options for the *fifth* character.

Applying the product rule:  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \underline{5!}$ .

# Permutations

“How many ways to order  $n$  distinct options?”

Answer:  $n! = n \cdot (n - 1) \cdot (n - 2) \cdots 1$

**$n$  factorial**

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 1$$

We only define  $n!$  for natural numbers  $n$ .

Note  $0! = 1$

# Example – Distinct letters 2

How many length 5 sequences are there consisting of distinct elements of  $\{A, B, C, \dots, Z\}$ ? **26**

Step 1: 26 options for the *first* element.

Step 2: 25(remaining) options for the *second* element.

Step 3: 24(remaining) options for the *third* element.

Step 4: 23(remaining) options for the *fourth* element.

Step 5: 22(remaining) options for the *fifth* element.

**=**

# Example – Distinct letters 2

How many length 5 sequences are there consisting of distinct elements of  $\{A, B, C, \dots, Z\}$  ?

Step 1: 26 options for the *first* element.

Step 2: 25 (remaining) options for the *second* element.

Step 3: 24 (remaining) options for the *third* element.

Step 4: 23 (remaining) options for the *second* element.

Step 5: 22 (remaining) options for the *third* element.

Applying the product rule:  $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$ .

# $k$ -permutation

## $k$ -permutation

length  $k$

The number of  $k$ -element sequences of distinct symbols from a universe of  $n$  symbols is:

$$P(n, k) = n \cdot (n - 1) \cdots (n - k + 1) = \frac{n!}{(n - k)!}$$

$n$  pick  $k$

$$26 \cdot 25 \cdot 24 \cdots 26 - 5 + 1 = 22 = \frac{26!}{21!} = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot \cancel{21!}}{\cancel{21!}}$$

# Number of Subsets

How many size-5 subsets are there consisting of distinct elements of  $\{A, B, C, \dots, Z\}$  ?

e.g.,  $\{A, Z, U, R, E\}$ ,  $\{B, I, N, G, O\}$ ,  $\{T, A, N, G, O\}$ . But not:  $\{R, A, I, N\}$ ,  $\{Z, Q, F\}$ ,...

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e.g.,  $\{A, Z, U, R, E\}$ ,  $\{B, I, N, G, O\}$ ,  $\{T, A, N, G, O\}$ . But not:  $\{R, A, I, N\}$ ,  $\{Z, Q, F\}$ ,...

idea:

$$\underline{\underline{(26-4)^5}}$$

TANGO

NTAGO

Different from  $k$ -permutations: NO ORDER

→ Different sequences: TANGO, OGNAT, ATNGO, NATGO, ONATG ...

Same set:  $\{T, A, N, G, O\}$ ,  $\{O, G, N, A, T\}$ ,  $\{A, T, N, G, O\}$ ,  $\{N, A, T, G, O\}$ ,  $\{O, N, A, T, G\}$ ...



# Number of Subsets – Fill in what we know

Consider the following process:

1. Choose an **unordered** subset  $S \subseteq \{A, B, \dots, Z\}$  of size  $|S| = 5$ . e.g.  $S = \{A, G, N, O, T\}$
2. Choose a permutation of letters in  $S$   
e.g., TANGO, AGNOT, NAGOT, GOTAN, GOATN, NGOAT, ...

$$\begin{array}{c} \rightarrow \boxed{???.} \\ \times \\ \boxed{5!} \end{array}$$

Outcome: An ordered sequence of 5 distinct letters from  $\{A, B, \dots, Z\}$

$$= \frac{26!}{21!}$$

☺ 26 letters

$$???. =$$

# Number of Subsets – Fill in what we know

Consider the following process:

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↳ 1

Outcome: An **ordered** sequence of 5 distinct letters from  $\{A, B, \dots, Z\}$

???

×

5!

=

26!

21!

$$??? = \frac{26!}{21! 5!} = 65780$$

# Number of Subsets - $k$ -combination

## $k$ -combination

The number of  $k$ -element subsets from a set of  $n$  distinct symbols is:

$$C(n, k) = \frac{P(n, k)}{k!} = \frac{n!}{k! (n - k)!} = \binom{n}{k}$$

Examples:

- Selecting  $k$  people from a group of  $n$  to form a team
- Choosing  $k$  courses from a set of  $n$  courses to take in fall quarter
- Select 3 winners among  $n$  participants (no ordering between winners) -  $C(n, 3)$

# Pause

Questions in combinatorics and probability are often dense. A single word can totally change the answer. Does order matter or not? Are repeats allowed or not? What makes two things "count the same" or "count as different"?

Let's look for some keywords

# Pause – looking for keywords!

How many length 5 sequences are there consisting of distinct elements of  $\{A, B, C, \dots, Z\}$  ?

Sequences implies that order matters –  $(A, B, C)$  and  $(B, A, C)$  are different.

Distinct implies that you can't repeat elements  $(A, B, A)$  doesn't count.

$\{A, B, C, \dots, Z\}$  is our "universe" – our set of allowed elements.

Looking for an ordered sequence of distinct elements from a larger set → permutation!

# One more counting technique!

How many length 5 strings are there over  $\{a, b, \dots, z\}$  with at least 1 a?

aaaaa  
baaaa  
abbba  
✓

bbbbbb  
⋮

↓ ↓ ↓ b  
— a — a  
a

one idea:

choose a, 5 positions  
choose 4-length string  
 $5 \cdot 26^4$  ?

X but this overcounts

# One more counting technique!

How many length 5 strings are there over  $\{a, b, \dots, z\}$  with at least 1 a?

*we can rephrase this as:* "How many length 5 strings are there that do not have no a's?

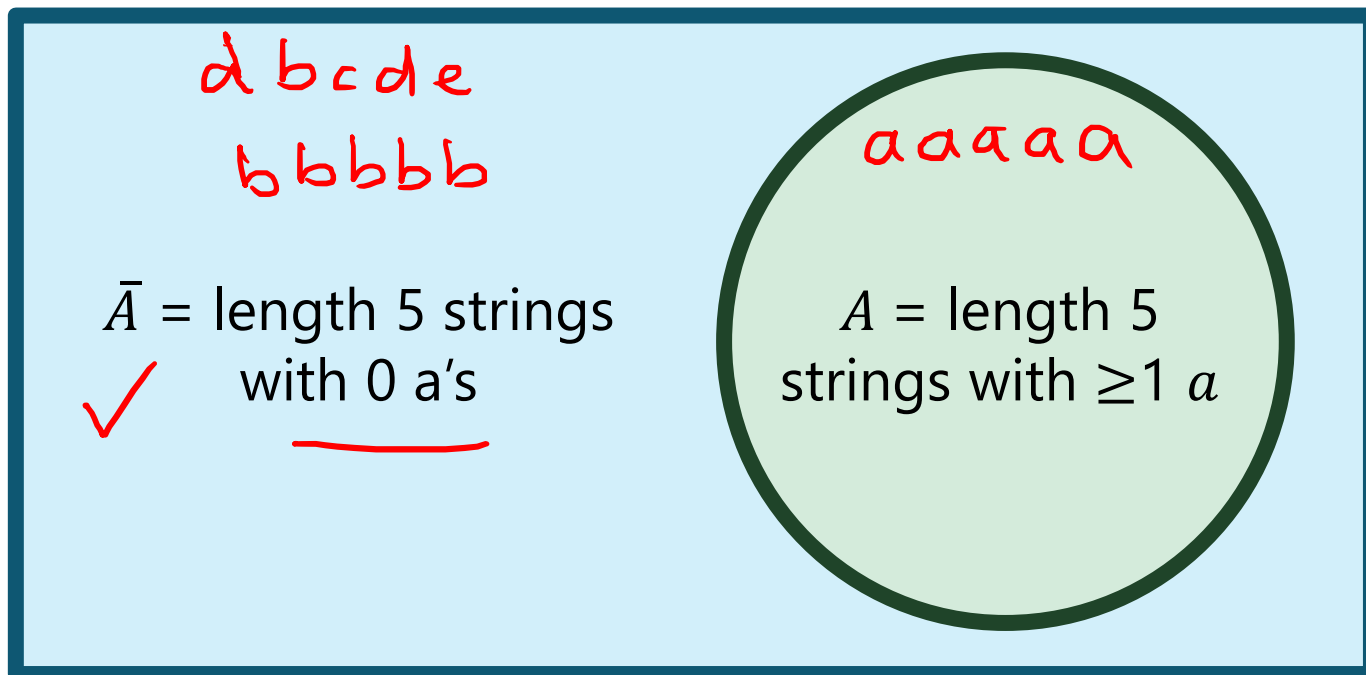
0 a's

# One more counting technique!

How many length 5 strings are there over  $\{a, b, \dots, z\}$  with **at least 1  $a$** ?

*we can rephrase this as:* "How many length 5 strings are there that do not have no  $a$ 's?"

$\mathcal{U}$  = all possible length 5 strings

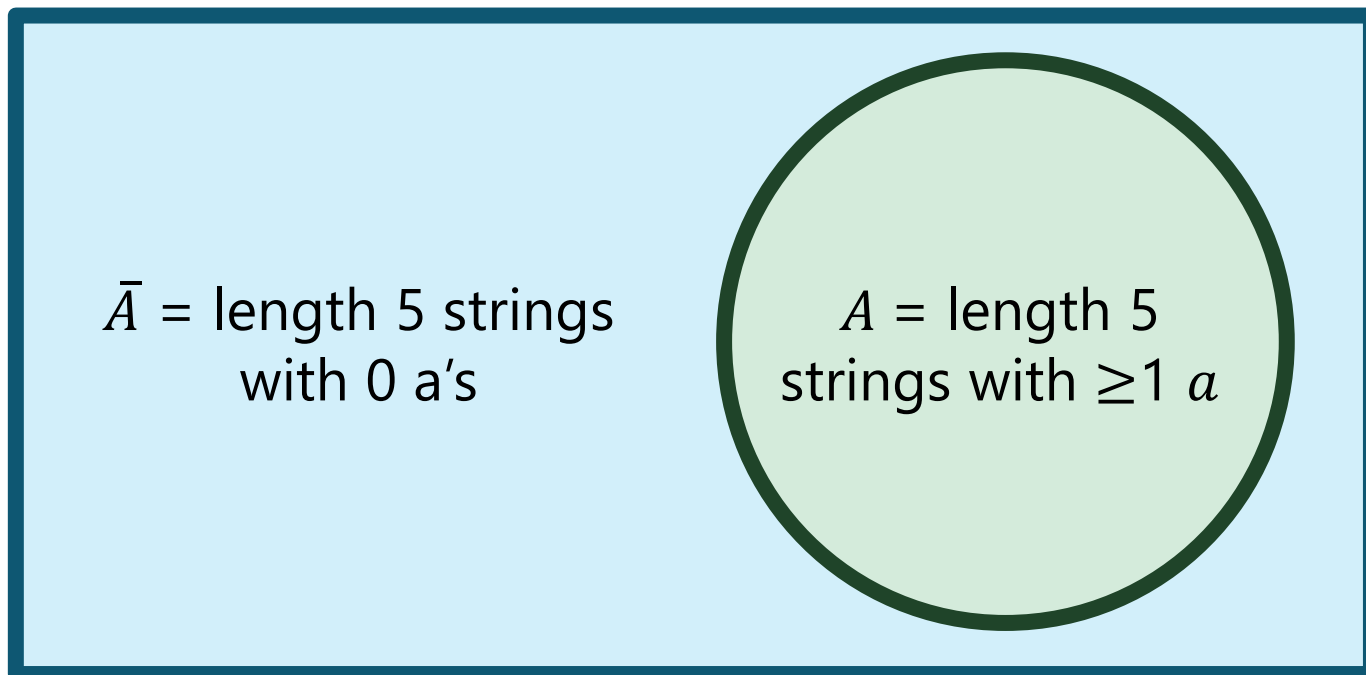


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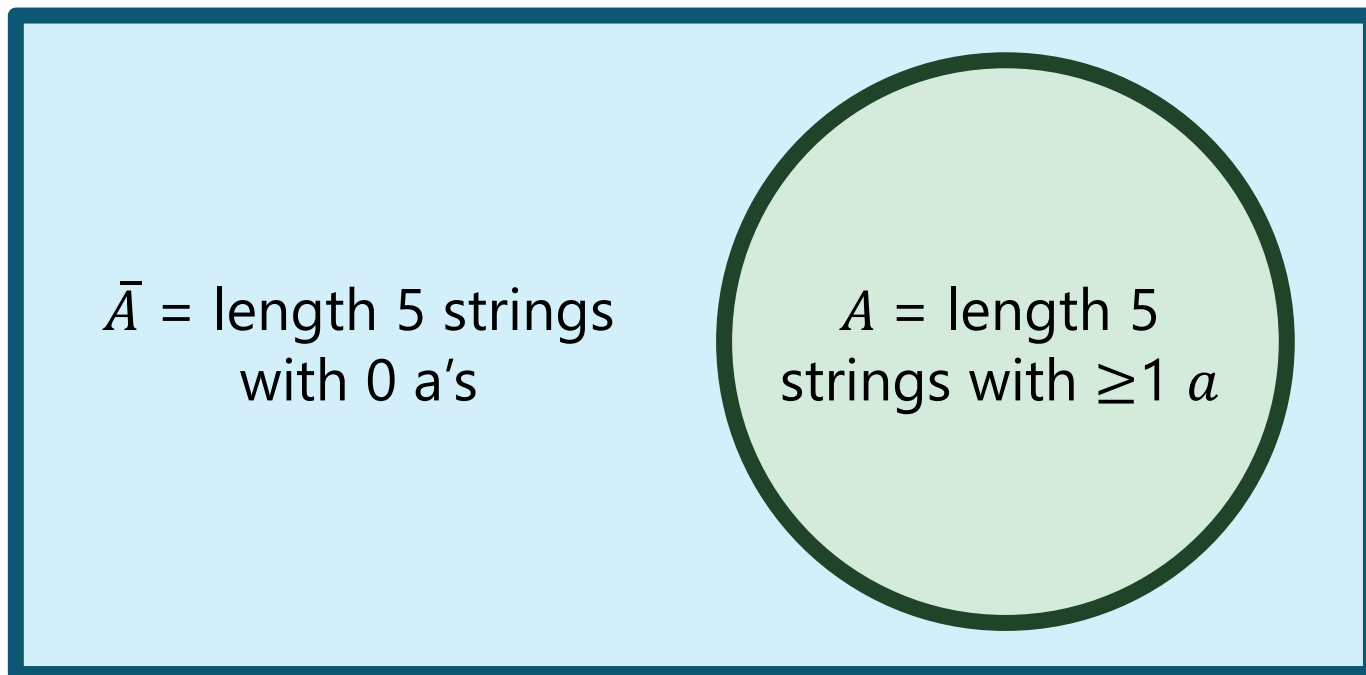
We're interested in  $|A|$  -

$$\begin{aligned} |A| &= |\mathcal{U} \setminus \bar{A}| = \underline{|\mathcal{U}|} - \underline{|\bar{A}|} \\ &= 26^5 - 25^5 \end{aligned}$$

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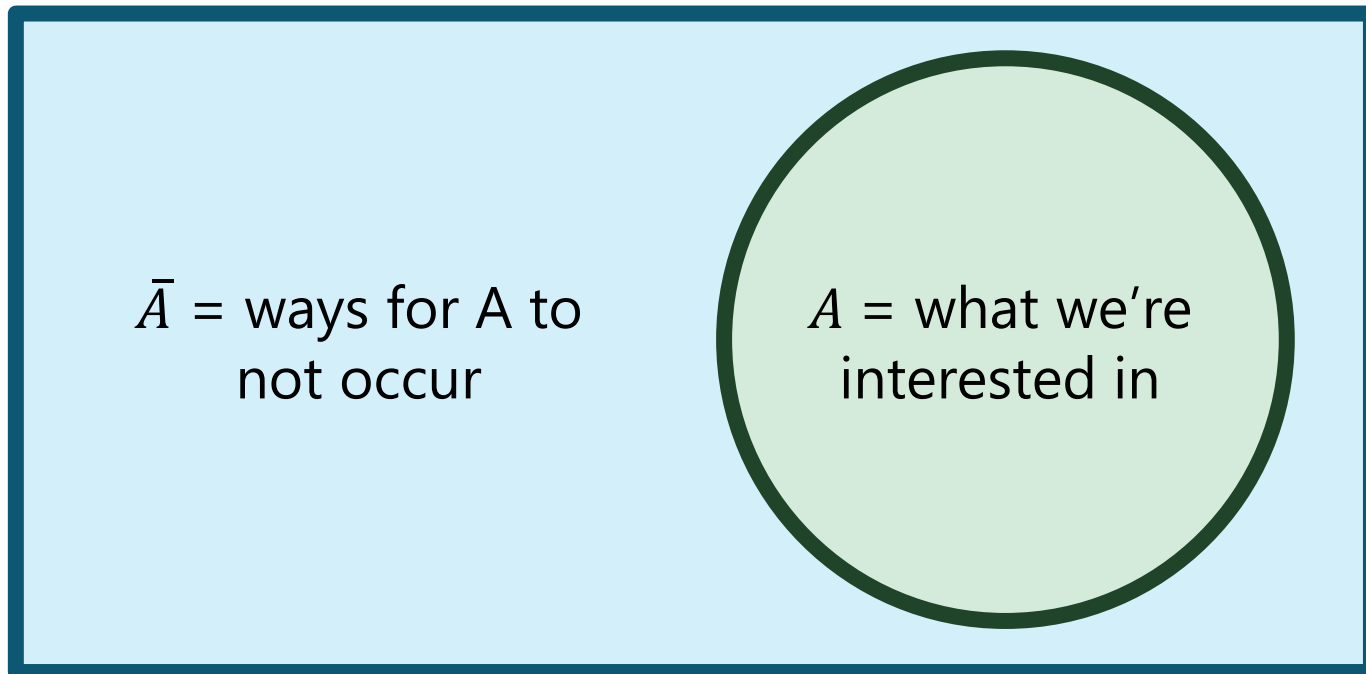
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# Complementary Counting

$$|A| = |\mathcal{U} \setminus \bar{A}| = |\mathcal{U}| - |\bar{A}|$$

total options – options for A to not occur = options for A to occur

$\mathcal{U}$  = all possible options



Phrases that *might* indicate complementary counting:

- ..options so that not \_\_ does occur
- ..ways for at least one \_\_ to occur = total – ways for none of \_\_ to occur

none, once

# That was a lot!!

"High level" counting approaches

- Sum rule – counts options where we have **disjoint** cases  $n + m$
- Product rule – counts options when we have a **sequential** process  $n_1 \cdot n_2 \cdots n_k$
- Complementary counting – we count the options by finding that total options and subtracting the non-desired options

*These may be used together! And may be used with the following --*

# That was a lot!!

**K-sequences:** How many length  $k$  sequences over alphabet of size  $n$ ?  
repetition allowed.

Product rule  $\rightarrow n^k$

**K-permutations:** How many length  $k$  sequences over alphabet of size  $n$ , without repetition?

Permutation  $\rightarrow \frac{n!}{(n-k)!}$

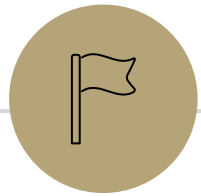
**K-combinations:** How many size  $k$  subsets of a set of size  $n$  (without repetition and without order)?

Combination  $\rightarrow \binom{n}{k} = \frac{n!}{k!(n-k)!}$

# TODOs from today

- ★ read through syllabus + agreement in Gradescope
- ★ make sure DRS accommodations are set up
- ★ email me with other accessibility concerns
- ★ first concept check will be released after class and due Friday 12pm
- ★ PSet1 will be released on Wednesday
- ★ Quiz 1 will be on Friday

P set 1 Q1 syllabus  
Q2 Math Review



**More Practice**

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# Baseball Outfits (setup)

The Husky baseball team has three hats (purple, black, gray)

Three jerseys (pinstripe, purple, gold)

And three pairs of pants (gray, white, black)

How many outfits are there (consisting of one hat, jersey, and pair of pants) if

the pinstripe jersey cannot be worn with gray pants,

the purple jersey cannot be worn with white pants,

and the gold jersey cannot be worn with black pants.

# Baseball Outfits (attempt 1)

Step 1: 3 choices for hats.

Step 2: 3 choices for jerseys

Step 3:...

# Baseball Outfits (answer 1)

Step 1: 3 choices for hats.

Step 2: 3 choices for jerseys.

Step 3: Regardless of which jersey we choose, we have 2 options for pants (even though there are three options overall).

$$3 \cdot 3 \cdot 2 = 18.$$