

Continuous Random Variables

CSE 312 Spring 26
Lecture 15

Agenda

- Continuous Random Variables
- Probability Density Function
- Cumulative Distribution Function
- Expectation and Variance of continuous r.v.
- Introduction to continuous zoo

Often we want to model experiments where the outcome is not discrete.

Definition. A **continuous random variable** X has a continuous range of values that it can take (an interval or a set of intervals).

Thus, a continuous random variable can take on an uncountable set of possible values

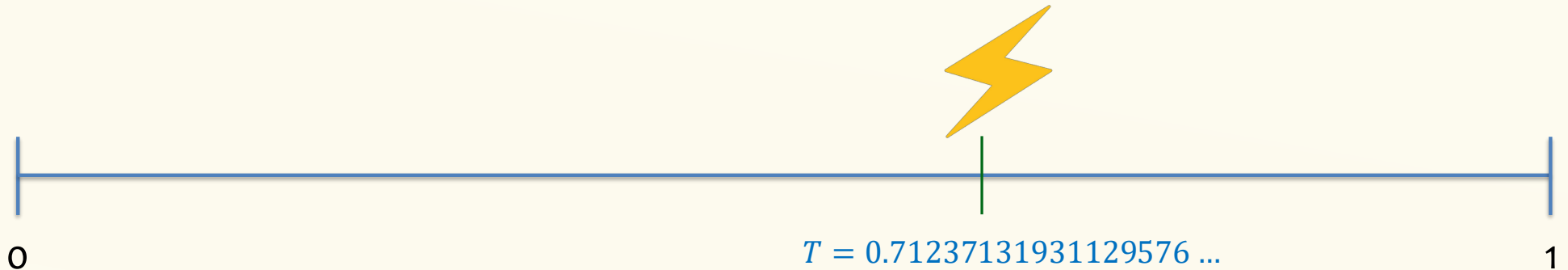
Examples:

- Time of an event
- Response time of a job
- Speed of a device
- Location of a satellite
- Distance between people's eyeballs

Example – Lightning Strike

Lightning strikes a pole within a one-minute time frame

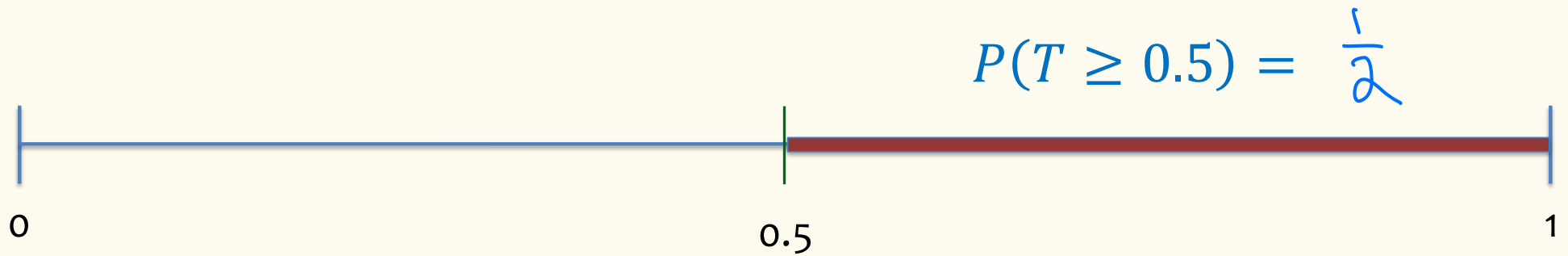
- T = time of lightning strike
- Every time within $[0,1]$ is equally likely
 - Time measured with infinitesimal precision.



The outcome space is not discrete

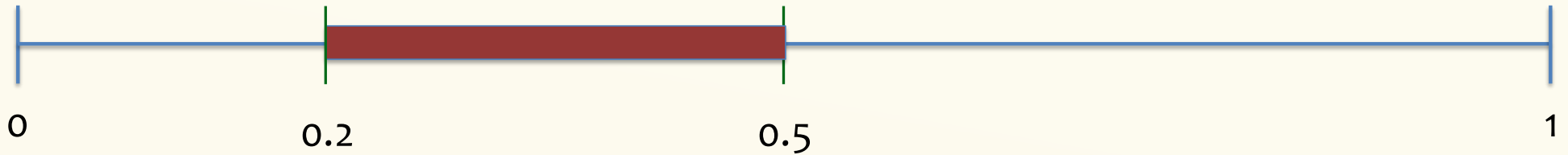
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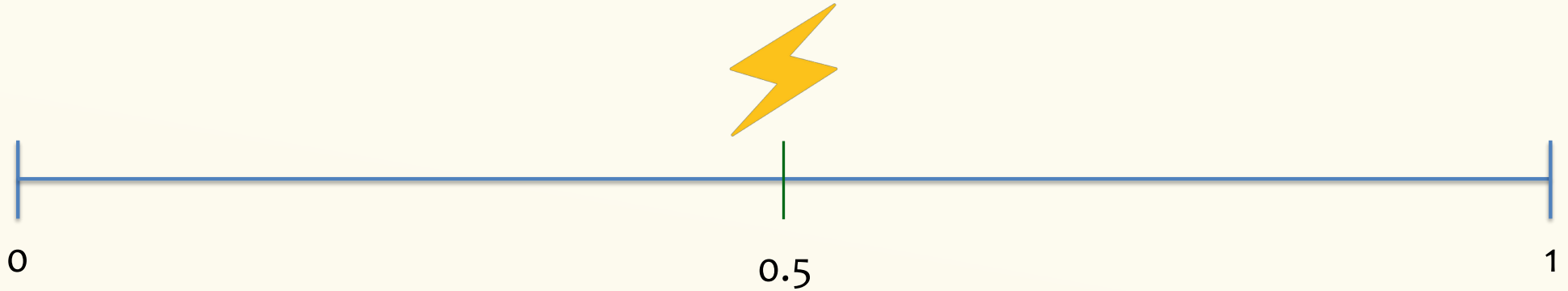
- T = time of lightning strike
- Every point in time within $[0,1]$ is equally likely



$$P(0.2 \leq T \leq 0.5) = 0.3$$

Lightning strikes a pole within a one-minute time frame

- T = time of lightning strike
- Every point in time within $[0,1]$ is equally likely



$$P(T = 0.5) = 0$$

Bottom line

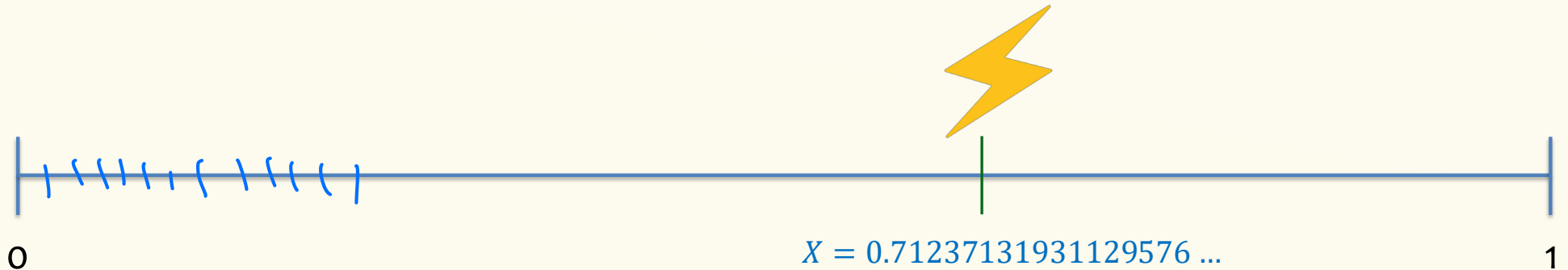
- This gives rise to a different type of random variable
- $P(T = x) = 0$ for all $x \in [0,1]$
- Yet, somehow we want
 - $P(T \in [0,1]) = 1$
 - $P(T \in [a, b]) = b - a$
 - ...
- How do we model the behavior of T ?

First try: A discrete approximation

Example – Lightning Strike

Lightning strikes a pole within a one-minute time frame

- X = time of lightning strike
- Every time within $[0,1]$ is equally likely
 - Time measured with infinitesimal precision.

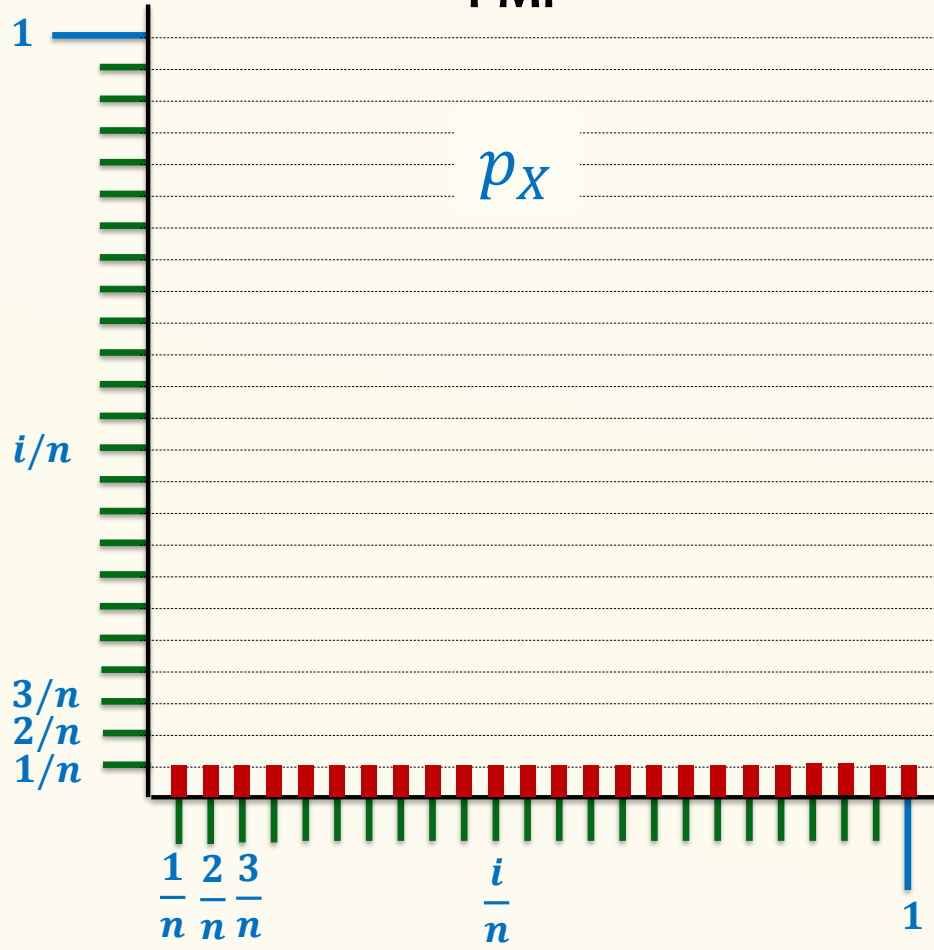


Discrete approximation?

A Discrete Approximation

Probability Mass Function

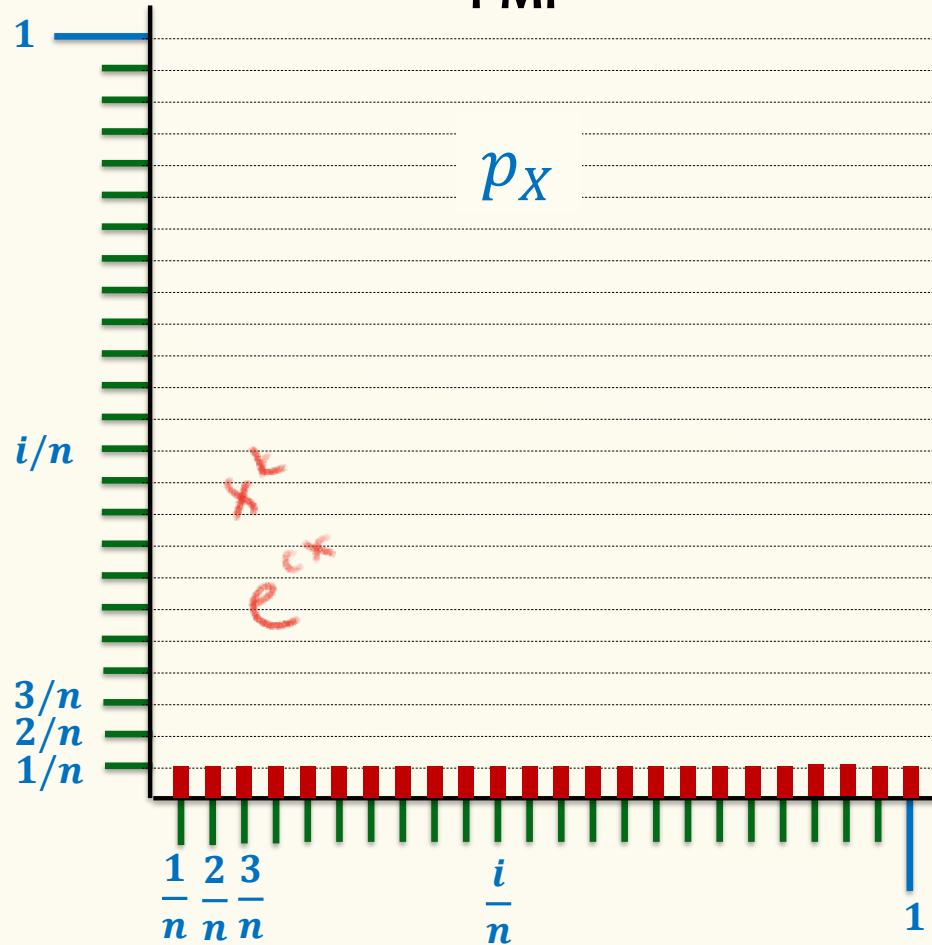
PMF



A Discrete Approximation

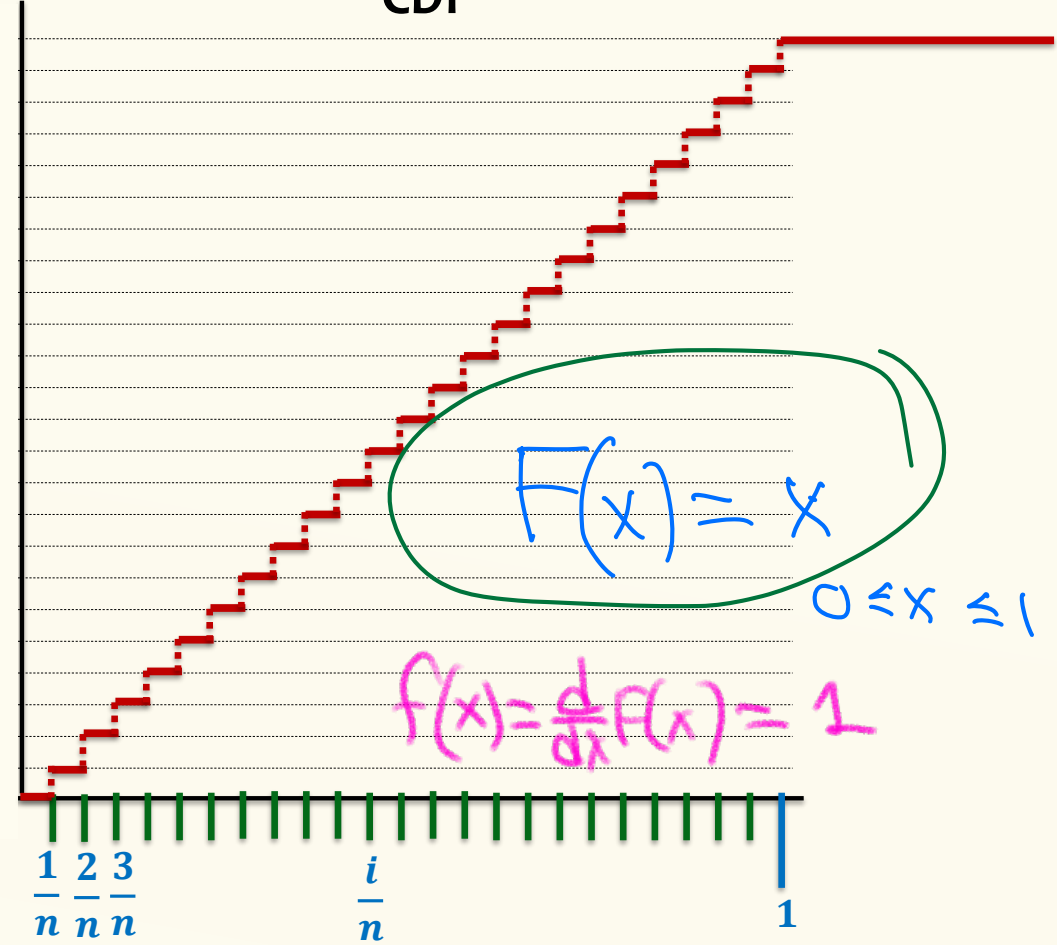
Probability Mass Function

PMF



Cumulative Distribution Function

CDF



$$P_X(x) = \frac{F_X(k) - F_X(k-1)}{k - (k-1)}$$

$$k-1 = x$$

$$k - (k-1) = dx$$

$$\frac{F_X(x+dx) - F_X(x)}{dx}$$

$$\lim_{dx \rightarrow 0} = \frac{d}{dx}$$

$F_X(x) = f_X(x)$ prob density fn.

$F_X(x)$ = total prob accumulated up to x

$f_X(x)$ = rate at which prob accumulates at x

Given the CDF F_X of a random variable X with $\Omega_X = \{0, 1, 2, \dots\}$, how do you compute the pmf?

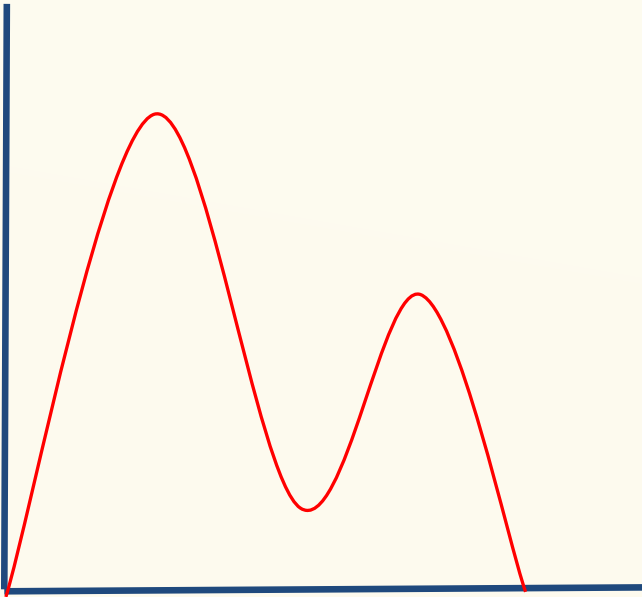
polllev.com/anamakarin185

$$\Pr(X = k) =$$

- a. $F_X(k - 1)$
- b. $F_X(1) + F_X(2) + \dots + F_X(k - 1)$
- c. $F_X(k) - F_X(k - 1)$
- d. I don't know.

Definition. A **continuous random variable** X is defined by a **probability density function** (PDF) $f_X: \mathbb{R} \rightarrow \mathbb{R}$, such that

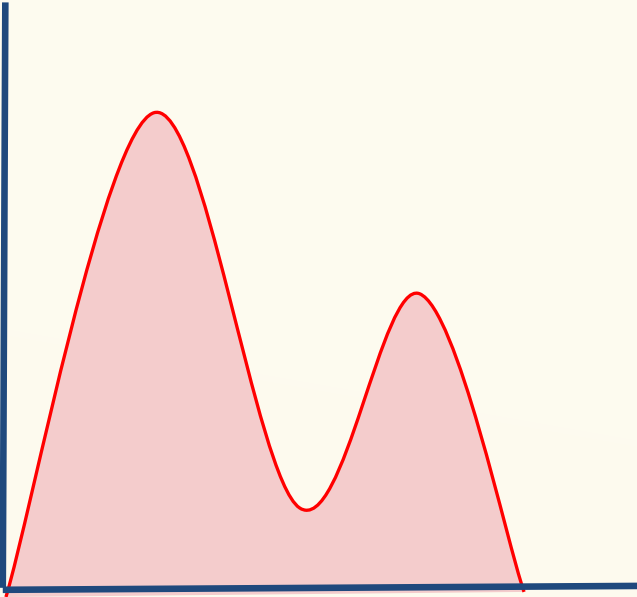
Non-negativity: $f_X(x) \geq 0$ for all $x \in \mathbb{R}$



Probability Density Function - Intuition

Non-negativity: $f_X(x) \geq 0$ for all $x \in \mathbb{R}$

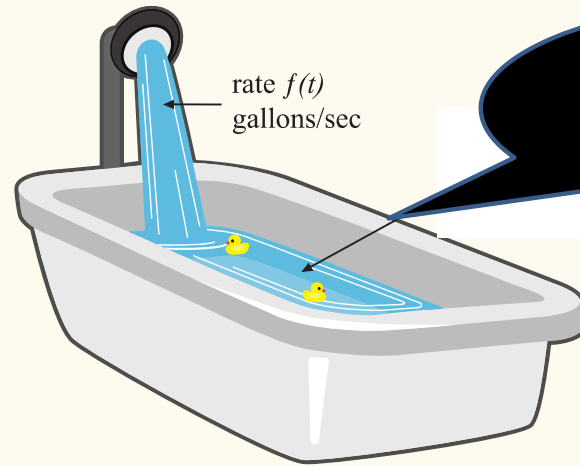
Normalization: $\int_{-\infty}^{+\infty} f_X(x) dx = 1$



Density as a rate

Density functions are not necessarily related to probability.

Example: Filling a bathtub at rate $f_X(t) = t^2$ gallons/sec, where $t \geq 0$

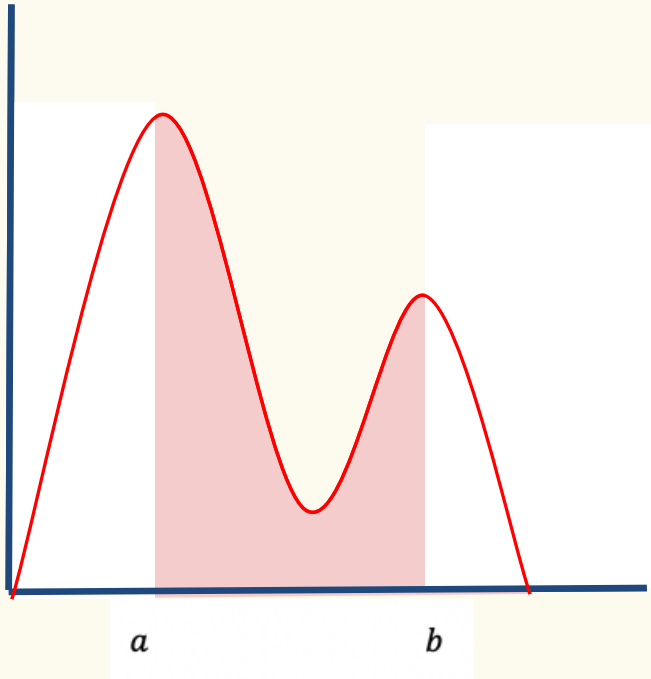


Q: How much water after 4 seconds?

A:
$$\int_0^4 f_X(t) dt = \int_0^4 t^2 dt = \frac{64}{3} \text{ gallons}$$

Q: Is $f_X(t)$ a p.d.f.?

Probability Density Function - Intuition

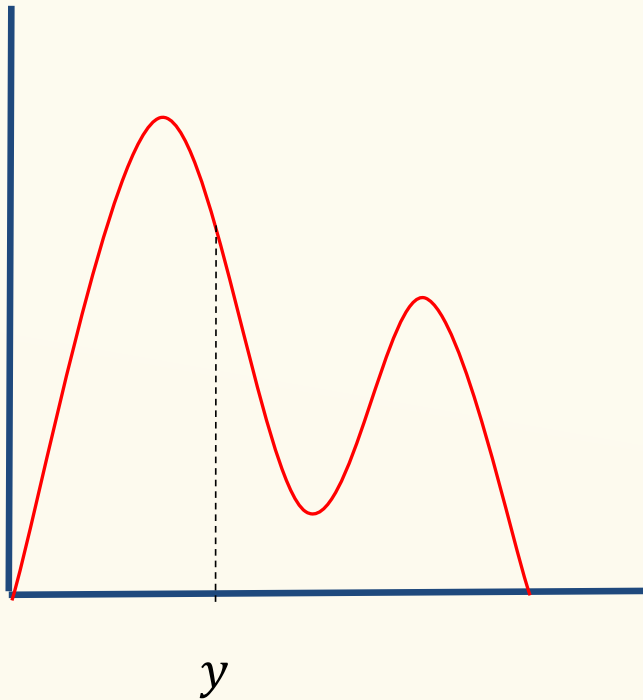


Non-negativity: $f_X(x) \geq 0$ for all $x \in \mathbb{R}$

Normalization: $\int_{-\infty}^{+\infty} f_X(x) dx = 1$

$$F(b) - F(a) = P(a \leq X \leq b) = \int_a^b f_X(x) dx$$

Probability Density Function - Intuition



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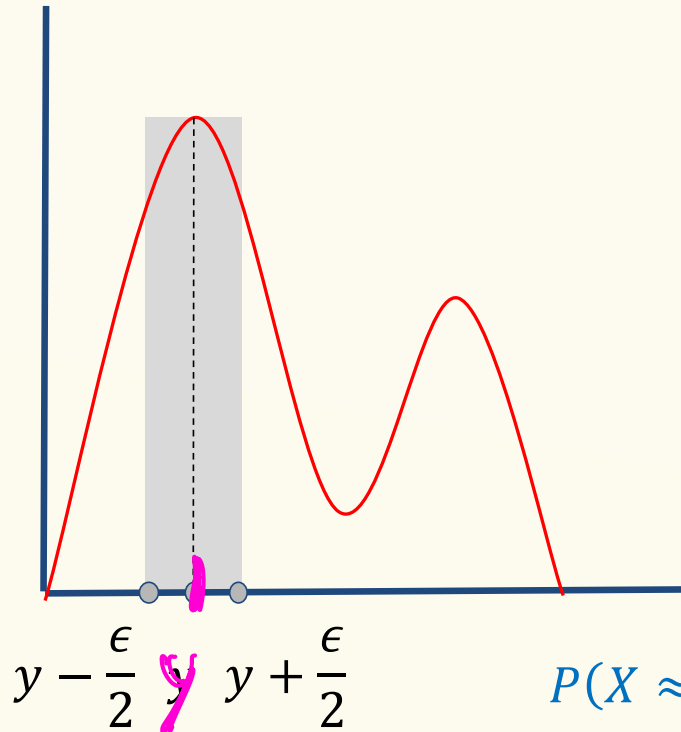
$$P(X = y) = P(y \leq X \leq y) = \int_y^y f_X(x) dx = 0$$



Density \neq Probability

$$f_X(y) \neq 0 \quad P(X = y) = 0$$

Probability Density Function - Intuition



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Normalization: $\int_{-\infty}^{+\infty} f_X(x) dx = 1$

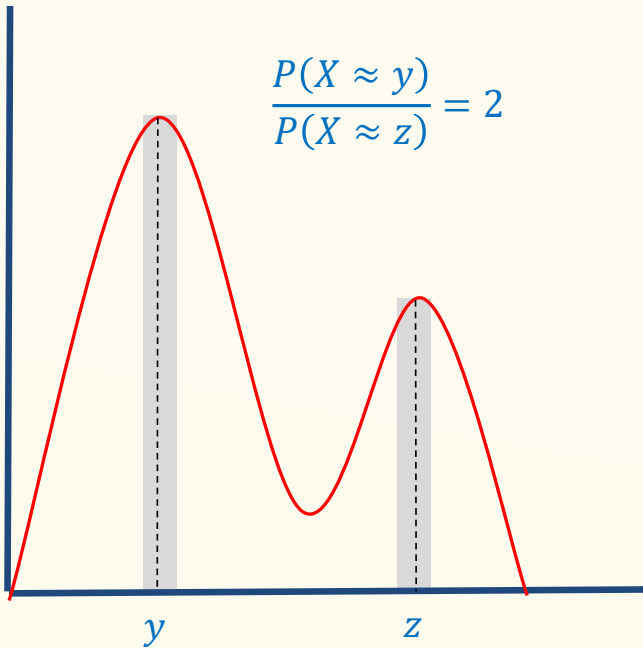
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$$P(X \approx y) \approx P\left(y - \frac{\epsilon}{2} \leq X \leq y + \frac{\epsilon}{2}\right) = \int_{y - \frac{\epsilon}{2}}^{y + \frac{\epsilon}{2}} f_X(x) dx \approx \epsilon f_X(y)$$

What $f_X(x)$ measures: The local **rate** at which probability accumulates

Probability Density Function - Intuition



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$$\frac{P(X \approx y)}{P(X \approx z)} \approx \frac{\epsilon f_X(y)}{\epsilon f_X(z)} = \frac{f_X(y)}{f_X(z)}$$

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Cumulative Distribution Function

Definition. The **cumulative distribution function (cdf)** of X is

$$F_X(a) = P(X \leq a) = \int_{-\infty}^a f_X(x) dx$$

By the fundamental theorem of Calculus $f_X(x) = \frac{d}{dx} F_X(x)$

Cumulative Distribution Function

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By the fundamental theorem of Calculus $f_X(x) = \frac{d}{dx} F_X(x)$

Therefore: $P(X \in [a, b]) = F_X(b) - F_X(a)$

F_X is monotone increasing, since $f_X(x) \geq 0$. That is $F_X(c) \leq F_X(d)$ for $c \leq d$

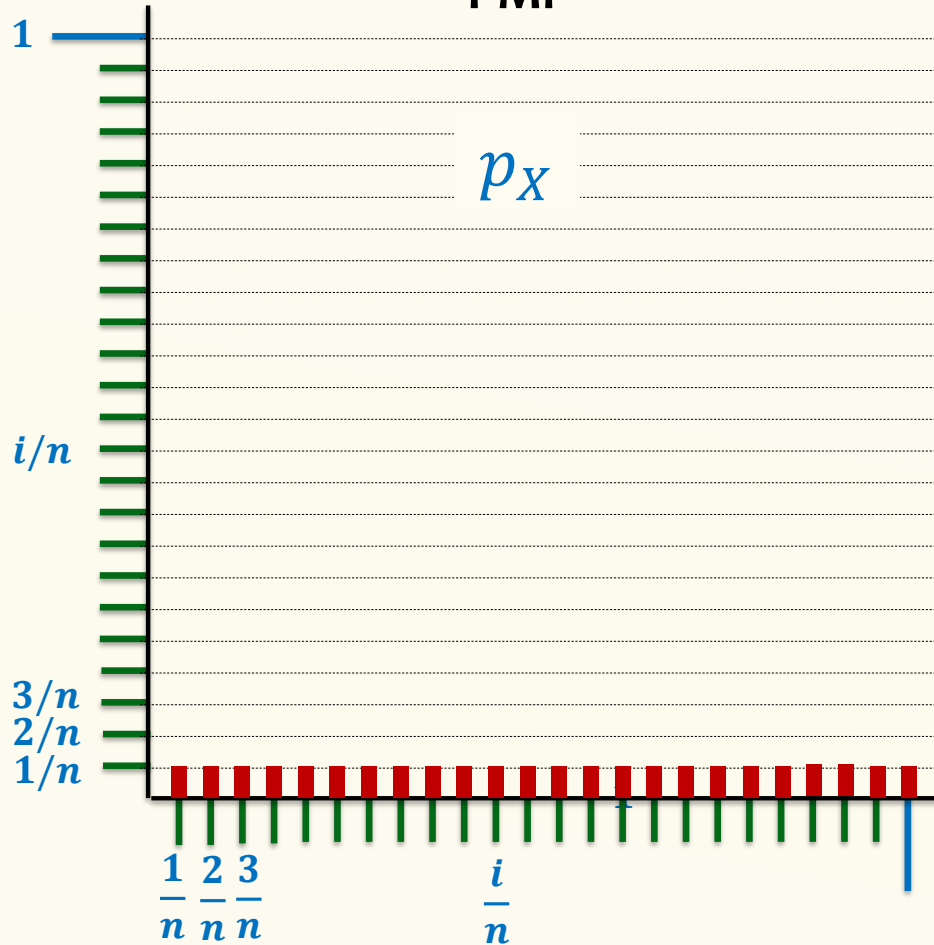
$$\lim_{a \rightarrow -\infty} F_X(a) = P(X \leq -\infty) = 0 \quad \lim_{a \rightarrow +\infty} F_X(a) = P(X \leq +\infty) = 1$$

From Discrete to Continuous

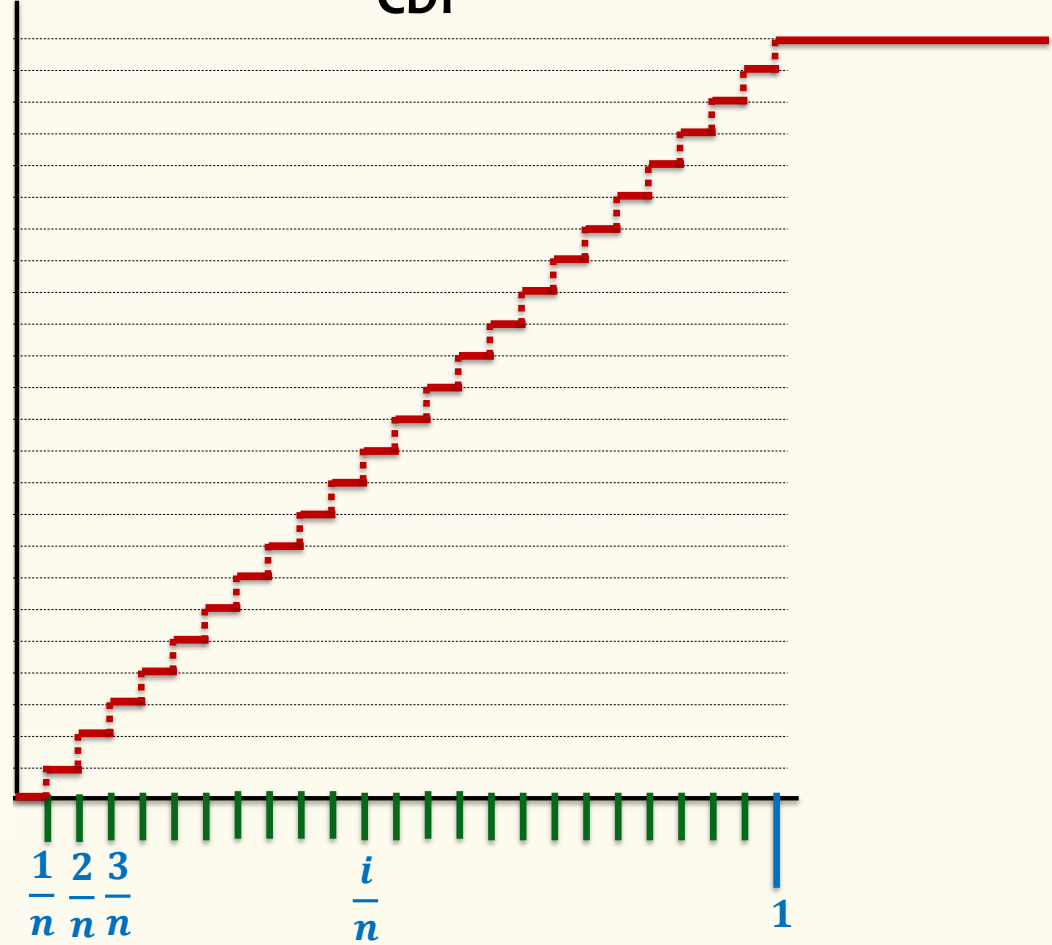
	Discrete	Continuous
PMF/PDF	$p_X(x) = P(X = x)$	$f_X(x) \neq P(X = x) = 0$
CDF	$F_X(x) = \sum_{t \leq x} p_X(t)$	$F_X(x) = \int_{-\infty}^x f_X(t) dt$
Normalization	$\sum_x p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x) dx = 1$

A Discrete Approximation

Probability Mass Function
PMF



Cumulative Distribution Function
CDF

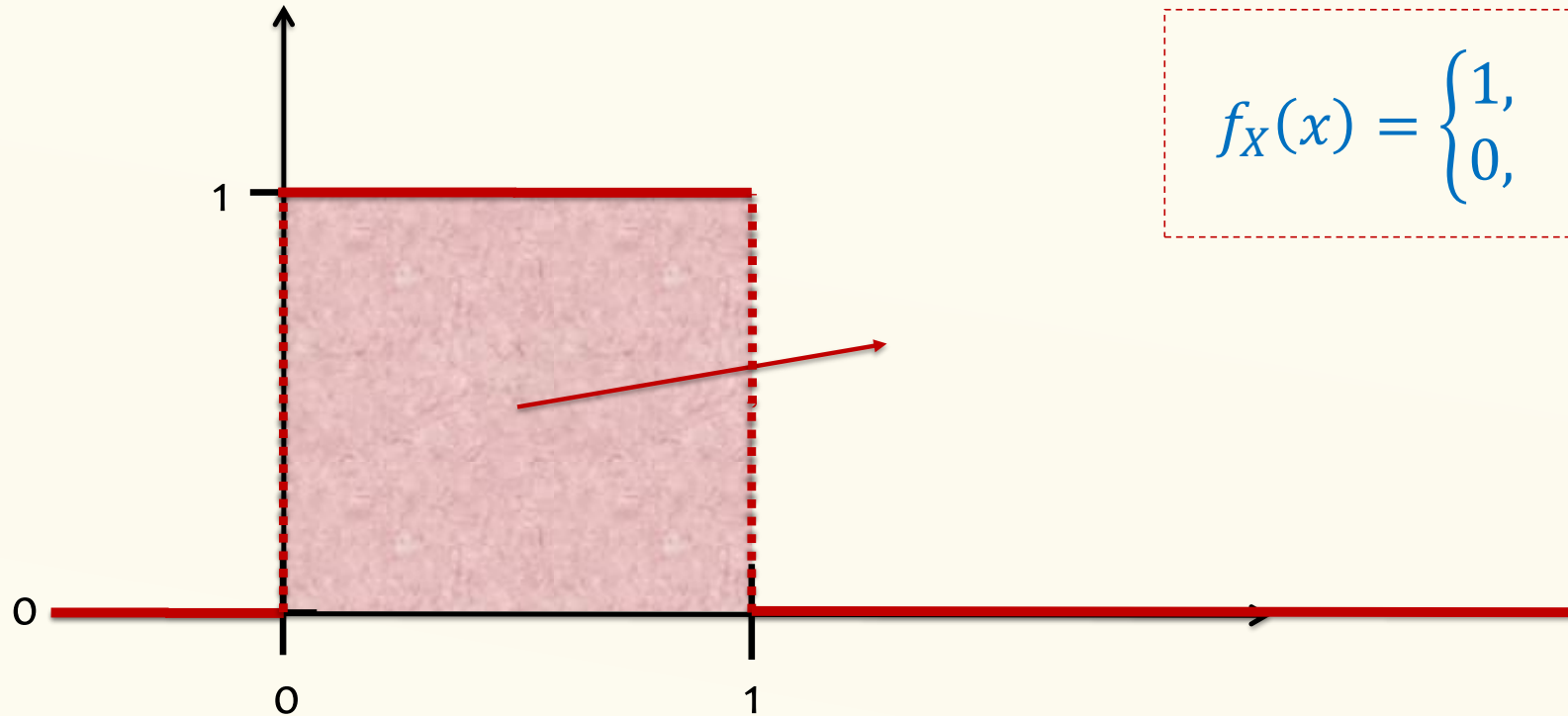


PDF of Uniform RV

$X \sim \text{Unif}(0,1)$

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x \leq 0 \\ x & 0 \leq x \leq 1 \\ 1 & 1 \leq x \end{cases}$$

$$f_X(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$



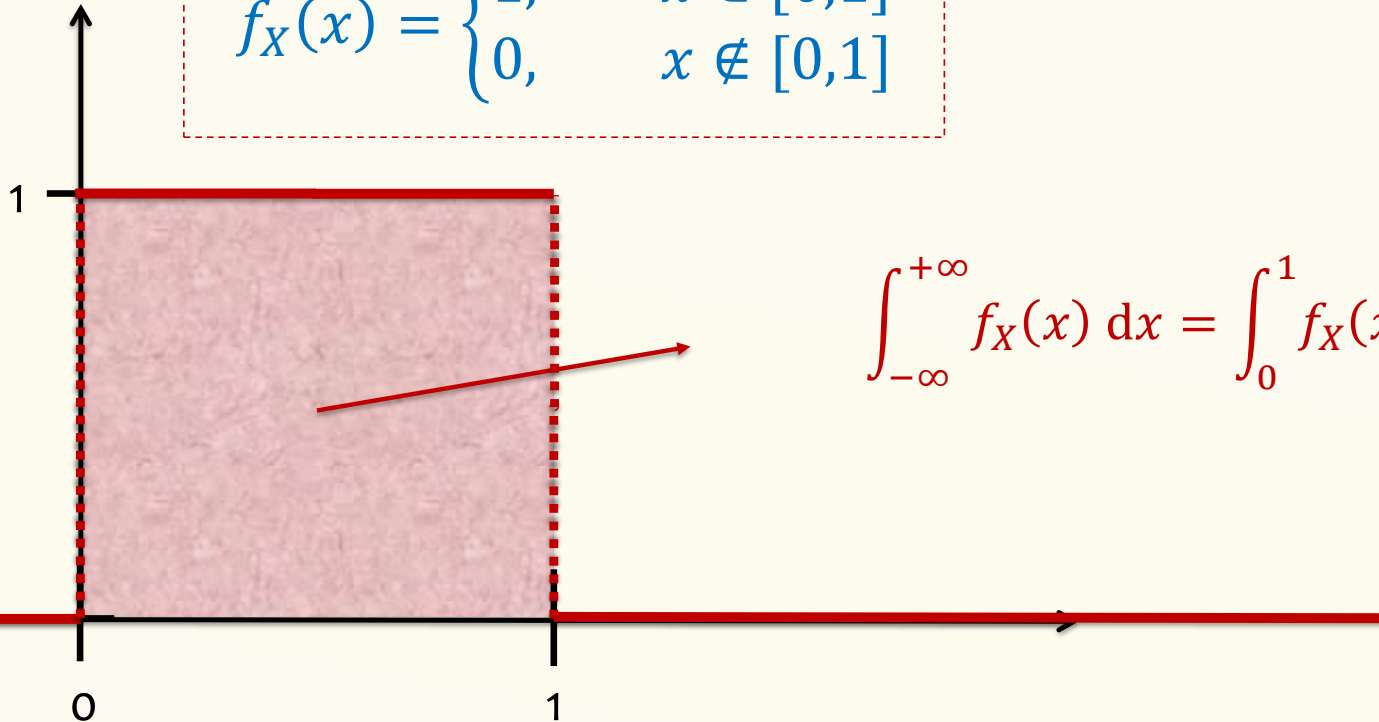
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$$f_X(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$



$$\int_{-\infty}^{+\infty} f_X(x) dx = \int_0^1 f_X(x) dx = 1 \cdot 1 = 1$$

Probability of Event

$X \sim \text{Unif}(0,1)$

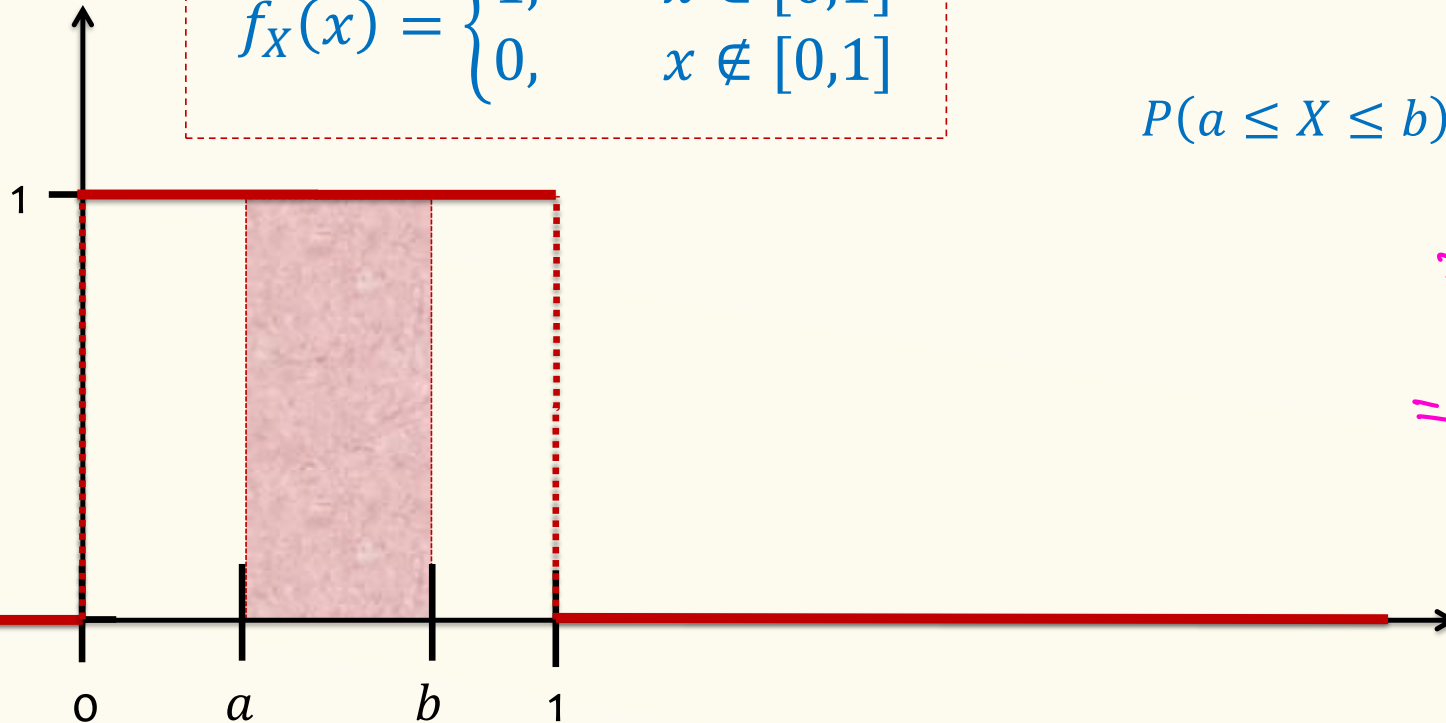
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Normalization: $\int_{-\infty}^{+\infty} f_X(x) dx = 1$

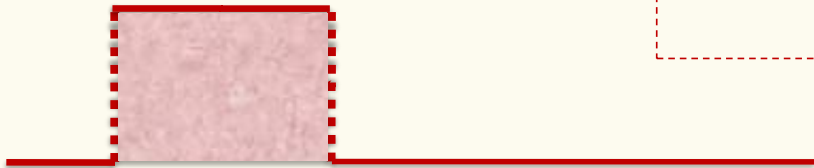
$$P(a \leq X \leq b) = \int_a^b f_X(x) dx$$

$$\begin{aligned} &= \int_a^b 1 dx \\ &= b - a \end{aligned}$$



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PDF of Uniform RV

$$X \sim \text{Unif}(0,0.5)$$

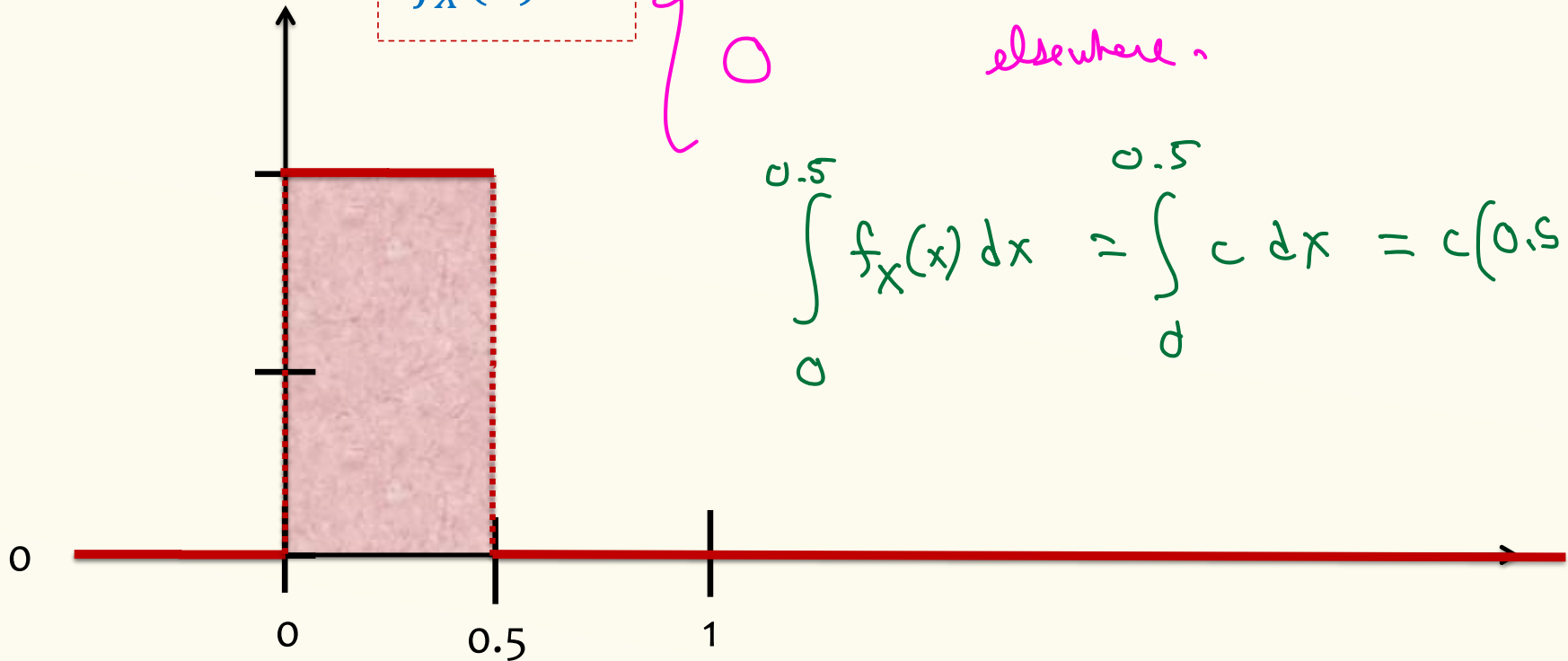
$$f_X(x) =$$

c
 0

$$0 < x < 0.5$$

elsewhere,

$$\int_0^{0.5} f_X(x) dx = \int_0^{0.5} c dx = c(0.5 - 0)$$



PDF of Uniform RV

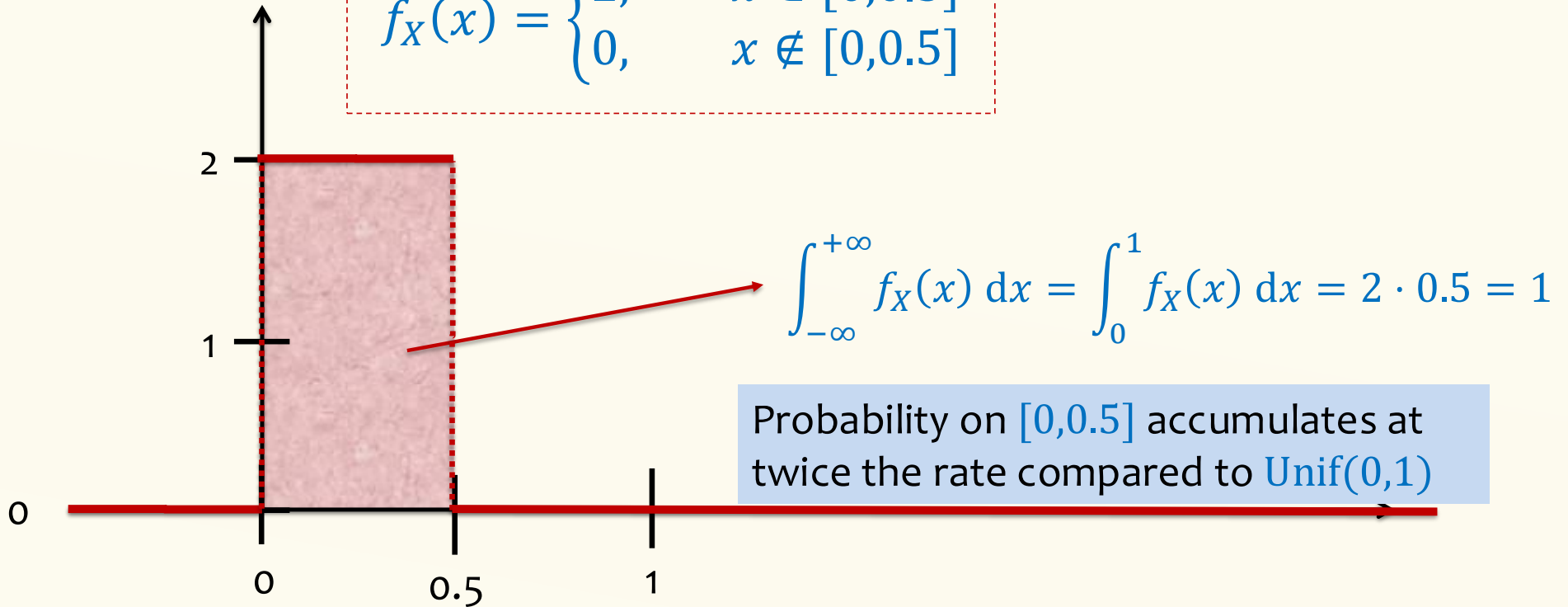
$$X \sim \text{Unif}(0,0.5)$$



Density \neq Probability

$f_X(x) \gg 1$ is possible!

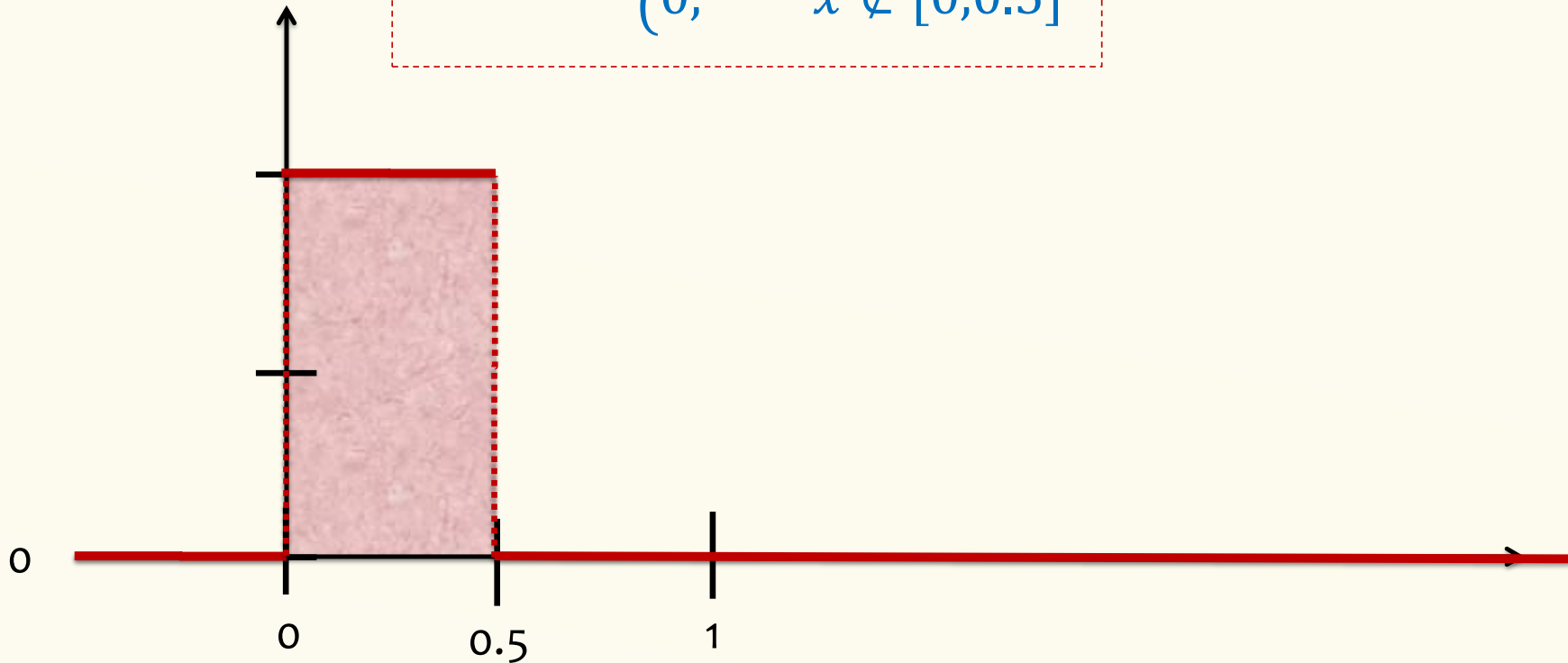
$$f_X(x) = \begin{cases} 2, & x \in [0,0.5] \\ 0, & x \notin [0,0.5] \end{cases}$$



PDF of Uniform RV

$X \sim \text{Unif}(0,0.5)$

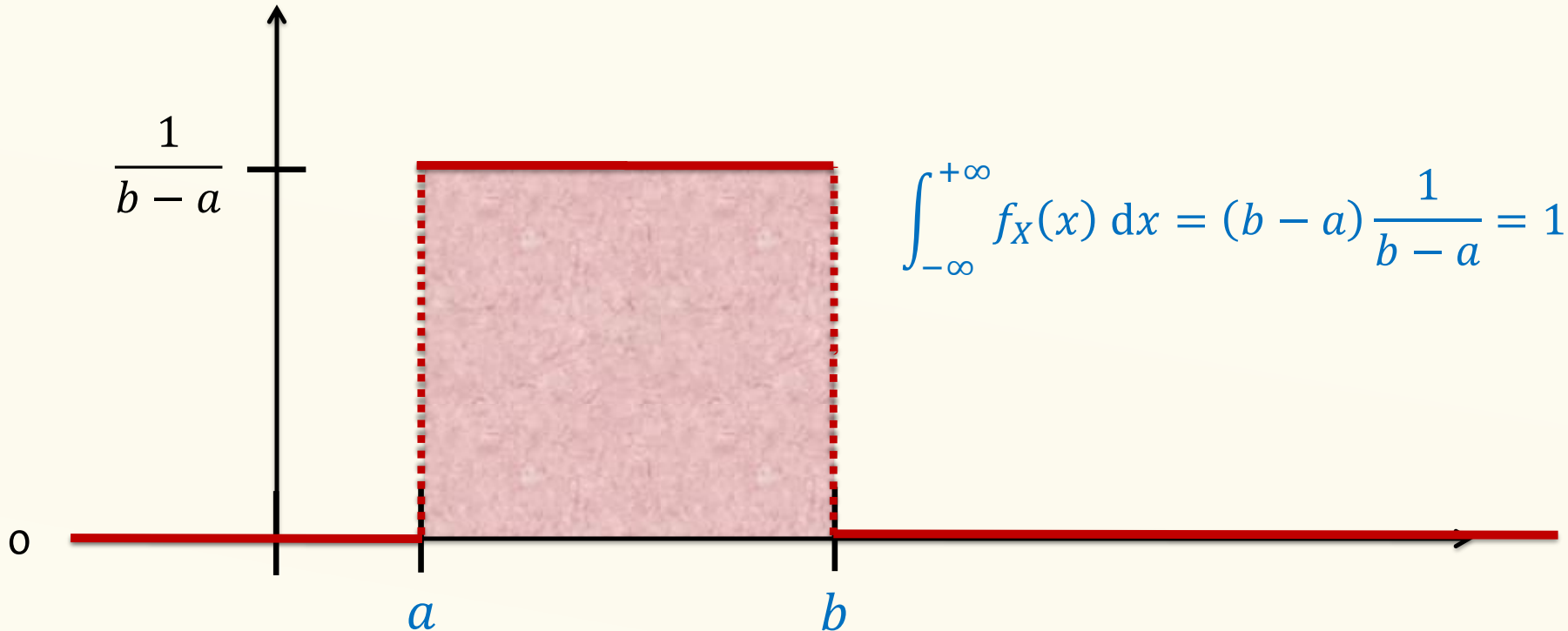
$$f_X(x) = \begin{cases} 2, & x \in [0,0.5] \\ 0, & x \notin [0,0.5] \end{cases}$$



Uniform Distribution


$X \sim \text{Unif}(a, b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$



$$\int_{-\infty}^{+\infty} f_X(x) dx = (b-a) \frac{1}{b-a} = 1$$

Agenda

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- Probability Density Function
- Cumulative Distribution Function
- Expectation and Variance of continuous r.v. 
- Introduction to continuous zoo

Expectation of a Continuous RV

Definition. The **expected value** of a continuous RV X is defined as

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx$$

Fact. $\mathbb{E}[aX + bY + c] = a\mathbb{E}[X] + b\mathbb{E}[Y] + c$

Proof follows same ideas as discrete case

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Proofs follow same ideas as discrete case

Definition. The **variance** of a continuous RV X is defined as

$$\text{Var}(X) = \int_{-\infty}^{+\infty} f_X(x) \cdot (x - \mathbb{E}[X])^2 \, dx = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\sum p_X(x)$$

Expectation, Variance, and LOTUS

Defn: For a continuous r.v. X with p.d.f. $f_X(\cdot)$, we have:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x) dx$$

LOTUS

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f_X(x) dx$$

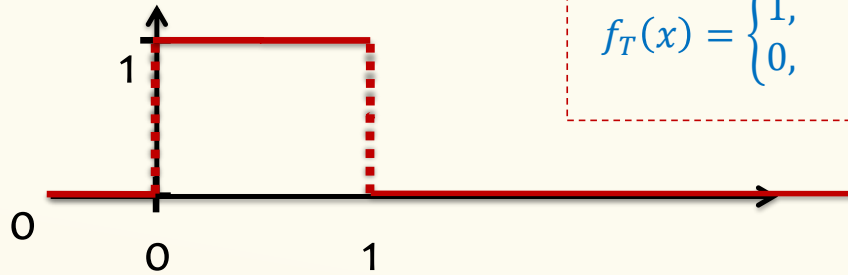
$$\mathbf{Var}(X) = \mathbf{E}[(X - \mathbf{E}[X])^2] = \int_{-\infty}^{\infty} (x - \mathbf{E}[X])^2 f_X(x) dx = \mathbf{E}[X^2] - \mathbf{E}[X]^2$$

Agenda

- Zoo of continuous random variables
 - Uniform Distribution
 - Exponential Distribution
 - Normal Distribution

Expectation of a Continuous RV

Example. $T \sim \text{Unif}(0,1)$



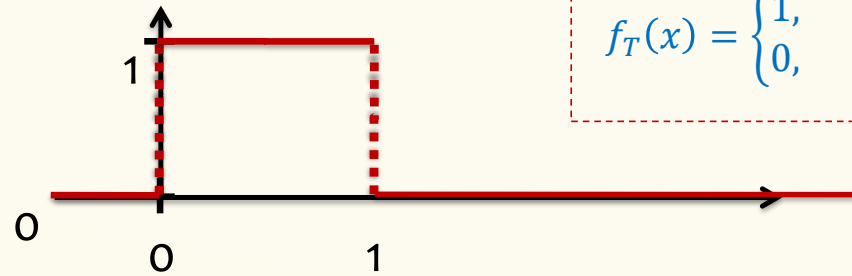
$$f_T(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$

Definition.

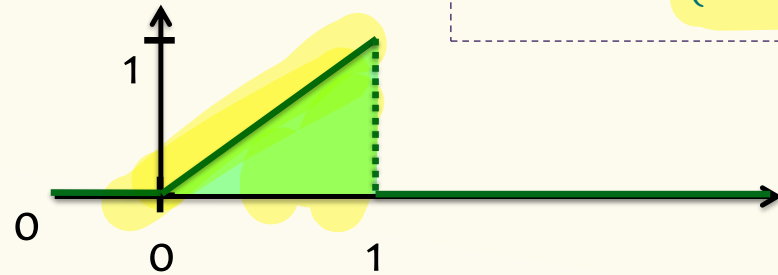
$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx$$

Expectation of a Continuous RV

Example. $T \sim \text{Unif}(0,1)$



$$f_T(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$



$$f_T(x) \cdot x = \begin{cases} x, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$

Definition.

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx$$

$$\mathbb{E}[X] = \underbrace{\frac{1}{2} 1^2}_{\text{Area of triangle}} = \frac{1}{2}$$

Area of triangle

Uniform Density – Expectation

$X \sim \text{Unif}(a, b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

$$\begin{aligned} \mathbb{E}[X] &= \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx \\ &= \frac{1}{b-a} \int_a^b x \, dx = \frac{1}{b-a} \left(\frac{x^2}{2} \right) \Big|_a^b = \frac{1}{b-a} \left(\frac{b^2 - a^2}{2} \right) \\ &= \frac{(b-a)(a+b)}{2(b-a)} = \frac{a+b}{2} \end{aligned}$$

Uniform Density – Variance

$$X \sim \text{Unif}(a, b)$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx$$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

Uniform Density – Variance

$X \sim \text{Unif}(a, b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{+\infty} f_X(x) \cdot x^2 \, dx$$

$$= \frac{1}{b-a} \int_a^b x^2 \, dx = \frac{1}{b-a} \left(\frac{x^3}{3} \right) \Big|_a^b = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{(b-a)(b^2 + ab + a^2)}{3(b-a)} = \frac{b^2 + ab + a^2}{3}$$

Uniform Density – Variance

$$\mathbb{E}[X^2] = \frac{b^2 + ab + a^2}{3} \quad \mathbb{E}[X] = \frac{a + b}{2}$$

$$X \sim \text{Unif}(a, b)$$

$$\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

Uniform Density – Variance

$$\mathbb{E}[X^2] = \frac{b^2 + ab + a^2}{3}$$

$$\mathbb{E}[X] = \frac{a + b}{2}$$

$$X \sim \text{Unif}(a, b)$$

$$\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{a^2 + 2ab + b^2}{4}$$

$$= \frac{4b^2 + 4ab + 4a^2}{12} - \frac{3a^2 + 6ab + 3b^2}{12}$$

$$= \frac{b^2 - 2ab + a^2}{12} = \frac{(b - a)^2}{12}$$

Uniform Distribution Summary

$X \sim \text{Unif}(a, b)$

pdf

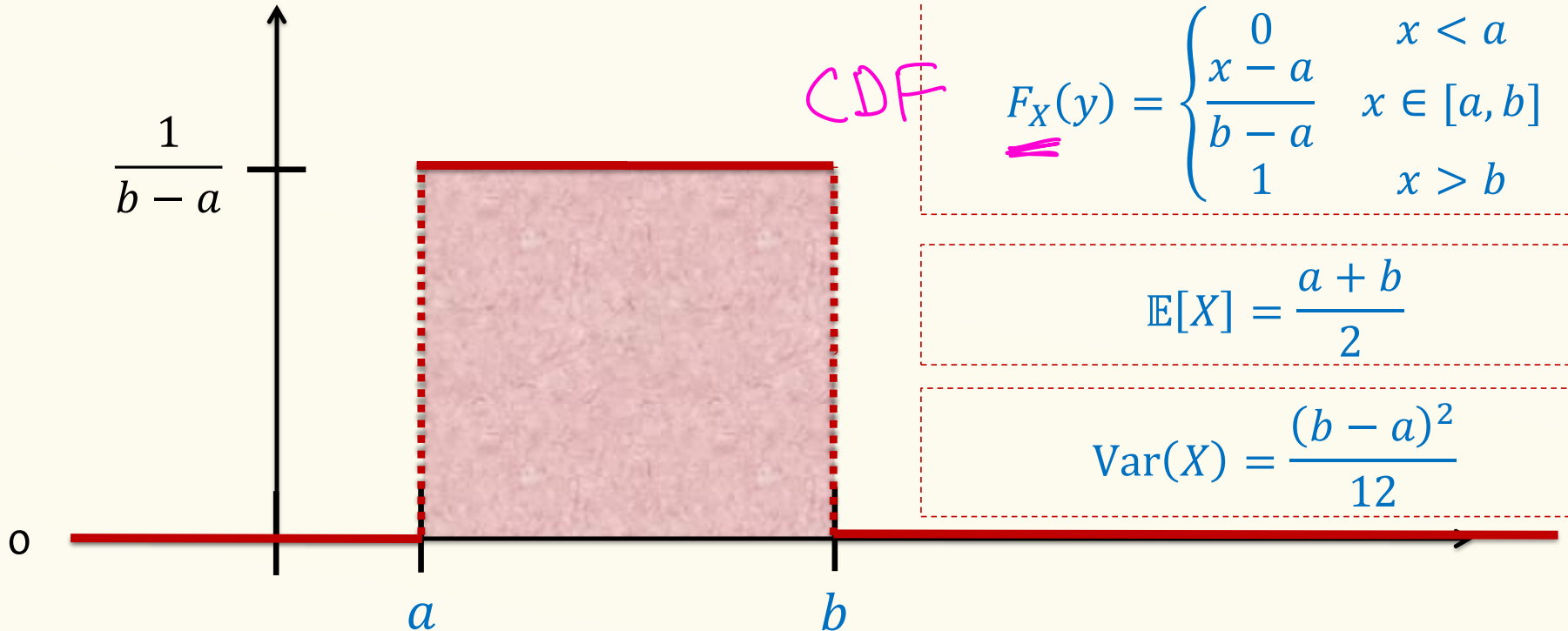
$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

CDF

$$F_X(y) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a, b] \\ 1 & x > b \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$



Agenda

- Zoo of continuous random variables
 - Uniform Distribution
 - Exponential Distribution

Recall Poisson

Assume expected # of occurrences of an event per unit of time is λ (independently)

- Cars going through intersection
- Number of lightning strikes
- Requests to web server
- Patients admitted to ER
- Rate of radioactive decay

Numbers of occurrences of event in one unit of time: Poisson distribution

$$P(W = i) = e^{-\lambda} \frac{\lambda^i}{i!} \quad (\text{Discrete})$$

How long to wait until next event? Exponential density!

Let's define it and then derive it!

Exponential Density - Warmup

$$W \sim \text{Poi}(\lambda) \Rightarrow P(W = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

Assume expected # of occurrences of an event per unit of time is λ (independently)

What is $\mathbb{E}[Z_t]$ where $Z_t = \#$ occurrences of event per t units of time?

$$E(Z_t) = \lambda t$$

$$Z_t \sim \text{Poi}(\lambda t)$$

X : time until first event

$$P(X > t) = P(\text{no event in first } t \text{ period of time}) \\ = P(Z_t = 0) = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-\lambda t}$$

$$F_X(t) = P(X \leq t) = \begin{cases} 1 - e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$f_X(t) = \lambda e^{-\lambda t} \quad t \geq 0$$

Exponential Density - Warmup

$$W \sim Poi(\lambda) \Rightarrow P(W = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

Assume expected # of occurrences of an event per unit of time is λ (independently)

What is the distribution of $Z_t = \#$ occurrences of event per t units of time?

$$\mathbb{E}[Z_t] = t\lambda$$

Z_t is independent over disjoint intervals

So $Z_t \sim Poi(t\lambda)$

$$W \sim Poi(\lambda) \Rightarrow P(W = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

The Exponential PDF/CDF

Assume expected # of occurrences of an event per unit of time is λ (independently)

Numbers of occurrences of event: Poisson distribution

How long to wait until next event? Exponential density!

- Let X be the time till the first event. We will compute $F_X(t)$ and $f_X(t)$
- We know $Z_t \sim Poi(t\lambda)$ is the # of events in the first t units of time, for $t \geq 0$.

$$W \sim Poi(\lambda) \Rightarrow P(W = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

The Exponential PDF/CDF

Assume expected # of occurrences of an event per unit of time is λ (independently)

Numbers of occurrences of event: Poisson distribution

How long to wait until next event? Exponential density!

- The exponential RV has range $[0, \infty]$, unlike Poisson with range $\{0, 1, 2, \dots\}$
- Let $X \sim Exp(\lambda)$ be the time till the first event. We will compute $F_X(t)$ and $f_X(t)$
- We know $Z_t \sim Poi(t\lambda)$ be the # of events in the first t units of time, for $t \geq 0$.
- $P(X > t) = P(\text{no event in the first } t \text{ units}) = P(Z_t = 0) = e^{-t\lambda} \frac{(t\lambda)^0}{0!} = e^{-t\lambda}$
- $F_X(t) = P(X \leq t) = 1 - P(X > t) = 1 - e^{-t\lambda}$
- $f_X(t) = \frac{d}{dt} F_X(t) = \lambda e^{-t\lambda}$

$$P(X > t) = e^{-t\lambda}$$

Exponential Distribution

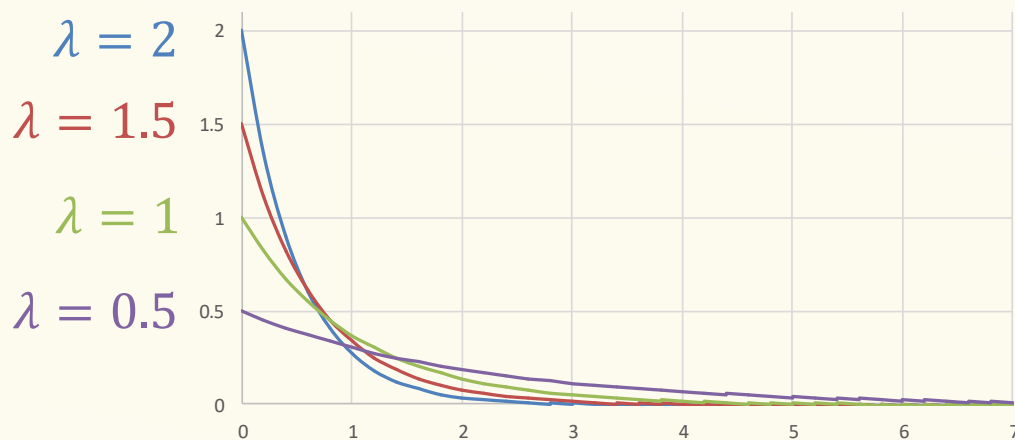
Definition. An **exponential random variable** X with parameter $\lambda \geq 0$ is follows the exponential density

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

We write $X \sim \text{Exp}(\lambda)$ and say X that follows the exponential distribution.

CDF: For $y \geq 0$,

$$F_X(y) = 1 - e^{-\lambda y}$$



Expectation

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx$$

$$= \int_0^{\infty} \lambda e^{-\lambda x} x \, dx$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$P(X > t) = e^{-t\lambda}$$

Expectation

$$\begin{aligned}\mathbb{E}[X] &= \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx \\ &= \int_0^{+\infty} \lambda e^{-\lambda x} \cdot x \, dx \\ &= \left(-\left(x + \frac{1}{\lambda}\right) e^{-\lambda x} \right) \Big|_0^{\infty} = \frac{1}{\lambda}\end{aligned}$$

Somewhat complex calculation
use integral by parts

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$P(X > t) = e^{-t\lambda}$$

$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

$$P(X > t) = e^{-t\lambda}$$

Exponential Distribution

We write $X \sim \text{Exp}(\lambda)$ and say X that follows the exponential distribution.

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CDF: For $y \geq 0$,
 $F_X(y) = 1 - e^{-\lambda y}$

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