CSE 312 – Problem Set 6

Due Wednesday, May 20, 11:59pm

Spring 2026

Instructions

Solutions format and late policy. See PSet 1 for details on expectations for solutions, collaboration policy, late policy, etc. The same requirements and policies still apply. Also follow the typesetting instructions from the prior PSets.

Solutions submission. You must submit your solution via Gradescope. In particular:

- Problem 1 will be done on Gradescope.
- Submit the solutions to problems 2-5 under "PSet 6 [Written]" as on previous problem sets. This will be a *single* PDF file containing the solution to problems 2-5 in the homework. Each numbered task should be solved on its own page (or pages). Follow the prompt on Gradescope to link tasks to your pages. Do not write your name on the individual pages – Gradescope will handle that.

1 Gradescope Questions (69 points)

We've got more questions than usual on gradescope (covering PDF/CDFs, exponential random variables, the CLT, etc); please do those here.

2 Charging Up (8 points)

Suppose you plug your electric vehicle into a public variable-rate charger to add exactly 180 miles of range. Suppose that the charging speed of this station (in miles of range added per hour) is uniformly distributed between 15 and 30 miles per hour. Let T be the time it takes to finish adding the 180 miles of range. Use LOTUS to compute $\mathbb{E}[T]$.

Hint: Recall that the indefinite integral of $g(x) = x^{-1}$ is equal to $\ln(x) + C$, assuming $x > 0$.

3 Alarms (11 points)

Suppose that you set three alarms to make sure that you get up in time for class.

- Alarm A goes off after X_A amount of time, where $X_A \sim \text{Exponential}(\lambda_A)$.
- Alarm B goes off after X_B amount of time, where $X_B \sim \text{Exponential}(\lambda_B)$.
- Alarm C goes off after X_C amount of time, where $X_C \sim \text{Exponential}(\lambda_C)$.

You may assume that each alarm is independent of one another.

a) (6 points) Let T be the time until the first of the three alarms go off. What is $\mathbb{P}(T \geq t)$? Use this to efficiently determine $\mathbb{E}[T]$ and $\mathrm{Var}(T)$.

b) (5 points) What is the probability that Alarm A is the one that wakes you up? That is, find $\mathbb{P}(X_A < X_B \cap X_A < X_C)$. Use the law of total probability, conditioning on the value of X_A.

One form of the continuous law of total probability is the following: If E is an event, and X is a continuous random variable with density function $f_X(x)$.

$$\mathbb{P}(E) = \int_{-\infty}^{\infty} \mathbb{P}(E \mid X = x) f_X(x) dx$$

The next two problems will require lookups in the Z-table.

4 Data from outer space (10 points)

Error-correcting codes are used in order to compensate for errors in transmission of messages (and in recovery of stored data from unreliable hardware). You are on a mission to Mars and need to send regular updates to mission control. Most of the packets actually don't get through, but you are using an error-correcting code that can let mission control recover the original message you send as long as at least 128 packets are received (not erased). Suppose that each packet gets erased independently with probability 0.6. How many packets should you send such that you can recover the message with probability at least 99%?

Use the Central Limit Theorem to approximate the answer, using the continuity correction. Final answer should be an integer number of packets and your intermediate calculations should be correct to sufficient precision (e.g. 4 decimal places) to ensure a good approximation.

5 Distant Stars (15 points)

An astronomer would like to measure the distance (in light years) from her observatory to a distant star. Each measurement is noisy, yielding only an estimate of the distance. Therefore, the astronomer plans to make a series of independent measurements and then use the average value of these measurements as her estimate of the actual distance. Suppose that each measurement has mean D (the true distance) and a variance of 4 light years. How many measurements does she need to make to be 95% confident that her estimate is accurate to within ± 0.5 light years?