Student name: ______________ **Student Number:** ______________

CSE 312: Foundations of Computing II - Autumn 2022

Midterm Exam - Practice

Important: Do not read the following pages until instructed to start. Read the instructions on this page carefully.

Instructions. This midterm is meant to be solved in 50 minutes. **The start time is your lecture start time and you are required to stop writing at** your lecture end time, unless otherwise instructed. When done, wait for a proctor to collect your solution, or follow any other instructions by the proctors.

- This exam consists of **five tasks**, overall with **100 points**.
- **Write your name and student number on top of this page**
- This is a **closed-book exam.** No written document is allowed. In particular, you are not allowed to use any textbooks, nor additional personal notes, homework assignments or solutions, etc.
- A detachable cheat sheet is provided on the front and back of the last page of this midterm.
- **No electronics are allowed** during the exam (no smart phones, no laptops, no smart watches, no pocket calculators, etc). Keep them stored in your bag.
- Write your solutions in the appropriate spaces (additional empty space is available at the end of the midterm – please reference to it if you use this space).

Good luck!

Task 1 – Short Questions [20 pts]

Are the following statements True or False? Provide a <u>short</u> justification for your answer, explaining why the statement is true or false.

a) $\binom{1024}{4} = \binom{1023}{4} + \binom{1023}{3}$.

b) Let X and Y be finite sets. Then, $|X \backslash Y| = |X| - |Y|$.

c) There exist events $\mathcal{A}$ and $\mathcal{B}$ (defined on the same probability space $(\Omega, \mathbb{P})$) such that $\mathbb{P}(\mathcal{A}) = 0.8$, $\mathbb{P}(\mathcal{B}) = 0.5$, and $\mathbb{P}(\mathcal{A} \cap \mathcal{B}) = 0.2$.

d) if A and B are events such that $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$ then A and B cannot be both independent and mutually exclusive.

e) If X and Y are random variables such that $\mathbb{E}[X+Y] = \mathbb{E}[X]+\mathbb{E}[Y]$ then X and Y are independent.

Task 2 – Counting [20 pts]

For both tasks, your final answers can contain factorials and/or binomial coefficients. However, do evaluate small factorials (up to $3! = 6$).

a) What is the number of re-arrangements of the word MIDTERM that contain no two consecutive vowels?

b) From a group of 20 people, we need to choose two committees – Committee A, of 5 people, and Committee B, of 7 people – which can share at most *one* member.

How many ways are there of doing so?

Task 3 – Basic Probability [15 pts]

A jar contains 3 red, 5 green, and 2 blue balls. Three times in a row, we pick a random ball (uniformly) from the jar, and then discard it (i.e., we do not add it back to the jar). Recall that this is referred to as sampling without replacement.

a) What is the probability that all balls are red?

b) What is the probability that all three balls have the same color?

c) What is the probability that both blue balls are chosen, along with a red ball?

Task 4 – Conditional Probabilities [20 pts]

Alice has two six-sided dice. The first die has all numbers between 1 and 6 on its sides. The second die has been manipulated, and only the even numbers 2, 4, 6 appear on its sides, i.e., each of the three numbers appears on *two* sides.

Assume that Alice picks the first die with probability $2/3$, and the second die with probability $1/3$. She then rolls the selected die, and we can assume that the die roll itself is fair, i.e., all sides are equally likely. We are interested in the number obtained from the roll of the chosen die.

a) What is the probability that Alice obtains an even number?

b) If Alice sees obtains an even number, what is the probability that the first die was rolled?

c) Are the events "an odd number is chosen" and "the second die is rolled" independent?

Task 5 – Red Balls [25 pts]

An urn contains 3 white and 3 red balls. Eve picks 2 balls from this urn at random *without* replacement. Let R be the number of red balls among the two balls she picks.

a) Give the PMF of R. (In particular, give the range Ω_R of R.)

b) Compute $\mathbb{E}[R]$ directly using **a)**.

c) Compute $\mathbb{E}[R]$ using the linearity of expectation and indicator random variables.

d) What is $\mathbb{E}\left[R^2\right]$? Simplify the result as far as possible.

e) What is $\mathsf{Var}(R)$? (Try to compute this as efficiently as possible.)

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