

Student name: _____

Student Number: _____

CSE 312: Foundations of Computing II -

Autumn 2022

Midterm Exam

Important: Do not read the following pages until instructed to start. Read the instructions on this page carefully.

Instructions. This midterm is meant to be solved in 50 minutes. **The start time is 1:30pm** and you are required to stop writing at **2:20pm**, unless otherwise instructed. When done, wait for a proctor to collect your solution, or follow any other instructions by the proctors.

- This exam consists of **five tasks**, overall with **100 points**.
- **Write your name and student number on top of this page**
- This is a **closed-book exam**. No written document is allowed. In particular, you are not allowed to use any textbooks, nor additional personal notes, homework assignments or solutions, etc.
- A detachable cheat sheet is provided on the front and back of the last page of this midterm. Please be gentle when removing it.
- **No electronics are allowed** during the exam (no smart phones, no laptops, no smart watches, no pocket calculators, etc). Keep them stored in your bag.
- Write your solutions in the appropriate spaces (additional empty space is available at the end of the midterm – **please add a pointer to where the rest of your solution is if you used any extra space**).

Good luck!

Task 1 – Short Questions

[20 pts]

Are the following statements True or False? Provide a short justification for your answer, explaining why the statement is true or false.

a) $\sum_{k=0}^n \binom{n}{k} 4^k = 5^n$.

b) If A , B , and C are events then $\mathbb{P}(A \cup B \cup C) \leq \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C)$.

c) Let A and B be events such that $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.6$ and $\mathbb{P}(A \cup B) = 0.9$. Are the events A and B independent?

- d) Even without knowing any actual birthdays, we can be certain that in our CSE 312 class, which has more than 150 students registered, there must be at least 22 who were born on the same day of the week.

- e) If X and Y are independent random variables, each taking values in $\{+1, -1\}$, then

$$\mathbb{E}[(X + Y)^2] = 2 + 2\mathbb{E}[X] \cdot \mathbb{E}[Y].$$

Task 2 – Counting

[15 pts]

For both tasks, your final answers can contain factorials and/or binomial coefficients. However, do evaluate small factorials (up to $3! = 6$).

a) A CSE 312 baseball team consists of 9 distinct positions.

How many ways are there to designate these distinct positions as either for a P (professor), T (teaching assistant), or S (student) if we require that the team consist of

- 1 professor,
- 3 teaching assistants (TAs), and
- 5 students?

b) Xinyue is ordering 20 lunches for a group lunch. There are 5 options for each lunch. Of these options, 3 are sandwiches, Chicken, Kafta, or Falafel, and 2 are salads, Greek or Tabbouleh. A lunch order consists of the numbers of each option it includes.

How many different lunch orders does Xinyue have to choose from?

Task 3 – Basic Probability

[20 pts]

A jar contains four red balls numbered 1 to 4, four blue balls numbered 1 to 4, and four green balls numbered 1 to 4. We choose three balls from the jar, one after another, uniformly at random, without replacing any after each choice.

a) What is the probability that the balls we chose were a red 1, a blue 2, and a green 3 in that order?

b) What is the probability that the three balls we chose include a red ball and a blue ball that have the same number?

c) What is the probability that not all balls are the same color?

Task 4 – Conditional Probabilities

[20 pts]

Percy's restaurant offers a chance at different discounts on dinner depending on the day of the week. At the end of the meal, each customer gets a random chance at either a Gold or Silver discount. (They may get no discount at all.)

There are three cases depending on the day of the week:

- A. On Sundays there is a $1/2$ chance of getting a Silver discount and a $1/2$ chance of getting a Gold discount.
- B. On Mondays or Tuesdays there is a $2/3$ chance of getting a Silver discount and a $1/4$ chance of getting a Gold discount.
- C. On Wednesdays, Thursdays, Fridays, or Saturdays there is a $1/3$ chance of getting a Silver discount and a $1/4$ chance of getting a Gold discount.

Suppose that Tanmay chose one of the 7 days of the week to go to Percy's restaurant uniformly at random:

a) What is the probability that Tanmay got a Gold discount?

b) If all you know is that Tanmay ended up with a Gold discount, what is the probability that Tanmay went to Percy's on a Sunday?

Task 5 – The Check Isn't in the Mail

[25 pts]

Paul is a bit old-fashioned and sometimes pays his bills by sending checks in the mail. He carefully writes out and signs each check. He did this for 3 bills this month: Tax, Water, and Gas. However, he was very distracted by the need to compose the CSE 312 midterm and didn't pay attention to which envelopes he was putting them in before sealing them up and putting them in the mail. They ended up scrambled, uniformly at random, one per envelope. Let X be the number of Paul's bills that got sent with the wrong check in them.

a) Give the PMF of X . (In particular, give the range Ω_X of X .)

b) Compute $\mathbb{E}[X]$ directly using **a**).

c) Compute $\mathbb{E}[X]$ using the linearity of expectation and indicator random variables.

d) What is $\mathbb{E}[X^2]$? Simplify the result as far as possible.

e) What is $\text{Var}(X)$? (Try to compute this as efficiently as possible.)

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