CSE 312 Section 7

Continuous RVs and CLT

Announcements & Reminders

- HW5
  - Grades released on gradescope – check your submission to read comments
  - Regrade requests open ~24 hours after grades are released and close after a week

- HW6
  - Released today
  - Due Wednesday 2/26 @ 11:59 pm
  - Late deadline Saturday 3/1 @ 11:59 pm

Review & Questions

Any lingering questions from this last week?

Each week in section, we'll be reviewing the main concepts from this week and putting them into action by going through some practice problems together. But before we get into that review, we'll try to start off each section with some time for you to ask questions. Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

Review of Main Concepts

- **Normal (Gaussian, "bell curve")**: $X \sim \mathcal{N}(\mu,\ \sigma^2)$ iff X has the following probability density function:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}, \quad x \in \mathbb{R}$$

  $\mathbb{E}[X] = \mu$ and $Var(X) = \sigma^2$. The "standard normal" random variable is typically denoted Z and has mean 0 and variance 1: if $X \sim \mathcal{N}(\mu,\ \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$. The CDF has no closed form, but we denote the CDF of the standard normal as $\Phi(z) = F_Z(z) = \mathbb{P}(Z \leq z)$. Note from symmetry of the probability density function about $z = 0$ that: $\Phi(-z) = 1 - \Phi(z)$.

- **Standardizing**: Let X be any random variable (discrete or continuous, not necessarily normal), with $\mathbb{E}[X] = \mu$ and $Var(X) = \sigma^2$. If we let $Y = \frac{X-\mu}{\sigma}$, then $\mathbb{E}[Y] = 0$ and $Var(Y) = 1$.

- **Closure of the Normal Distribution**: Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Then, $aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$. That is, linear transformations of normal random variables are still normal.

Review of Main Concepts

- **"Reproductive" Property of Normals**: Let $X_1, \ldots, X_n$ be independent normal random variables with $\mathbb{E}[X_i] = \mu_i$ and $Var(X_i) = \sigma_i^2$. Let $a_1, \ldots, a_n \in \mathbb{R}$ and $b \in \mathbb{R}$. Then,

$$X = \sum_{i=1}^{n}(a_i X_i + b) \sim \mathcal{N}\left(\sum_{i=1}^{n}(a_i\mu_i + b), \sum_{i=1}^{n} a_i^2\sigma_i^2\right)$$

  There's nothing special about the parameters – the important result here is that the resulting random variable is still normally distributed.

- **Law of Total Probability (Continuous):** A is an event, and X is a continuous random variable with density function $f_X(x)$.

$$\mathbb{P}(A) = \int_{-\infty}^{\infty} \mathbb{P}(A|X = x) f_X(x) dx$$

Review of Main Concepts

- **Central Limit Theorem (CLT)**: Let $X_1, \ldots, X_n$ be iid random variables with $\mathbb{E}[X_i] = \mu$ and $Var(X_i) = \sigma^2$. Let $X = \sum_{i=1}^{n} X_i$, which has $\mathbb{E}[X] = n\mu$ and $Var(X) = n\sigma^2$. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, which has $\mathbb{E}\left[\overline{X}\right] = \mu$ and $Var(\overline{X}) = \frac{\sigma^2}{n}$. $\overline{X}$ is called the *sample mean*. Then, as $n \to \infty$, $\overline{X}$ approaches the normal distribution $\mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$. Standardizing, this is equivalent to $Y = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ approaching $\mathcal{N}(0,1)$. Similarly, as $n \to \infty$, X approaches $\mathcal{N}(n\mu, n\sigma^2)$ and $Y' = \frac{X-n\mu}{\sigma\sqrt{n}}$ approaches $\mathcal{N}(0,1)$.

  It is no surprise that $\overline{X}$ has mean μ and variance σ^2/n – this can be done with simple calculations. The importance of the CLT is that, for large n, regardless of what distribution X_i comes from, $\overline{X}$ is *approximately normally distributed with mean μ and variance σ^2/n*. Don't forget the continuity correction, only when $X_1, \ldots, X_n$ are discrete random variables.

- If $Z \sim N(0,1)$ $---P(Z \leq 0.42) = \Phi(0.42) = 0.66276$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884	0.71226	0.71566	0.71904	0.7224
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.7549
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337	0.77637	0.77935	0.7823	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493	0.87698	0.879	0.881	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189

Review Questions

Question #1

a) True or False: For any random variable $X, P(X = 5) = P(X - 5 = 0)$

- True
- False

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- True
- False

Question #2

b) True or False: For some continuous random variable X, $P(X \leq 5) \neq P(X < 5)$

- True
- False

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- True
- False

Question #3

c) True or False: Let $X \sim \mathcal{N}(\mu, \sigma^2)$ and $a, b \in \mathbb{R}$. Then $aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

- True
- False

Question #3

c) True or False: Let $X \sim \mathcal{N}(\mu, \sigma^2)$ and $a, b \in \mathbb{R}$. Then $aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

- True
- False

Question #4

e) Select one: Suppose we have n independent and identically distributed $X_1, X_2, \ldots, X_n$, each with mean μ and variance σ^2. Let $X = \sum_{i=1}^{n} X_i$. Then as n grows large, the Central Limit Theorem tells us that X behaves similarly to which normal distribution.

- $X \sim \mathcal{N}(n\mu, n\sigma^2)$
- $X \sim \mathcal{N}(\mu, n\sigma^2)$
- $X \sim \mathcal{N}(n\mu, \sigma^2)$
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Problem 3 – Normal Questions

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a) Let X be a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute $P(4 < X < 16)$.
b) Let X be a normal random variable with mean 5. If $P(X > 9) = 0.2$, approximately what is $Var(X)$
c) Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $P(X > c) = 0.1$

Work on this with the people around you and then we'll go over it together!

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Problem 3 – Normal Questions

- (a) Let X be a normal random with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute $P(4 < X < 16)$.

  Let $\frac{X-10}{6} = Z$. (Recall standardization: $Z \sim N(0,1)$)

  $$P(4 < X < 16) = P\left(\frac{4-10}{6} < \frac{X-10}{6} < \frac{16-10}{6}\right) = P(-1 < Z < 1) = \Phi(1) - \Phi(-1) \approx 0.68268$$

- (b) Let X be a normal random variable with mean 5. If $P(X > 9) = 0.2$, approximately what is $Var(X)$?

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- (b) Let X be a normal random variable with mean 5. If $P(X > 9) = 0.2$, approximately what is $Var(X)$?

  Let $\sigma^2 = \mathrm{Var}(X)$. Then $P(X > 9) = P\left(\frac{X-5}{\sigma} > \frac{9-5}{\sigma}\right) = 1 - \Phi\left(\frac{4}{\sigma}\right) = 0.2$

  From the phi table, we get that $\frac{4}{\sigma} = 0.845$. Therefore, $\sigma \approx 4.73 \Rightarrow \sigma^2 \approx 22.4$.

Problem 3 – Normal Questions

c) Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $P(X > c) = 0.1$

$$P(X > c) = P\left(\frac{X-12}{2} > \frac{c-12}{2}\right) = 1 - \Phi\left(\frac{c-12}{2}\right) = 0.1$$

So, $\Phi\left(\frac{c-12}{2}\right) = 0.9$. Looking up the Phi values in reverse lets us undo the Φ function, and gives us $\frac{c-12}{2} = 1.29$. Solving for c we get $c \approx 14.58$.

Problem 4 – Do it in Reverse

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a) Let X be a normal random variable with parameters $\mu = 8$ and $\sigma^2 = 9$. Find x such that $P(X \leq x) = 0.6$
b) Lots of statistics (like standardized test scores or heights) use percentiles to give context to where outcomes fall in a distribution. The nth percentile marks the outcome at which n% of the data points are less than the outcome. Let Y be a normal random variable with parameters $\mu = 15$ and $\sigma^2 = 4$. What value y marks the 85$^{\text{th}}$ percentile? What value b marks the 15$^{\text{th}}$ percentile?

Work on this with the people around you and then we'll go over it together!

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Let $\frac{X-8}{3} = Z$. By the scale and shift properties of normal random variables, $Z \sim \mathcal{N}(0,1)$. Thus, we must find z such that $P(Z \leq z) = 0.6$.

$$\Phi(z) = P(Z \leq z) = 0.6$$

$$\Phi^{-1}\left(\Phi(z)\right) = \Phi^{-1}(0.6)$$

Thus, $Z \approx 0.25$ by looking up the phi values in reverse to undo the Φ function. Then $\frac{x-8}{3} = z \approx 0.25$, so $x \approx 8.75$.

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- (b) Lots of statistics (like standardized test scores or heights) use percentiles to give context to where outcomes fall in a distribution. The nth percentile marks the outcome at which n% of the data points are less than the outcome. Let Y be a normal random variable with parameters $\mu = 15$ and $\sigma^2 = 4$. What value y marks the 85th percentile? What value b marks the 15th percentile?

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We first find y, which marks the 85th percentile, so $P(Y \le y) = 0.85$. Let $\frac{Y-15}{2} = Z$. By the scale and shift properties of normal random variables, $Z \sim N(0,1)$. Thus, we must find z such that $P(Z \le z) = 0.85$.

$$\Phi(z) = P(Z \le z) = 0.85$$
$$\Phi^{-1}\left(\Phi(z)\right) = \Phi^{-1}(0.85)$$

Thus, $z \approx 1.04$ by looking up the phi values in reverse to undo the Φ function.

Then $\frac{y-15}{2} = z \approx 1.04, so\ y \approx 17.08$.

Problem 4 – Do it in Reverse

- (b) Lots of statistics (like standardized test scores or heights) use percentiles to give context to where outcomes fall in a distribution. The nth percentile marks the outcome at which n% of the data points are less than the outcome. Let Y be a normal random variable with parameters $\mu = 15$ and $\sigma^2 = 4$. What value y marks the 85th percentile? What value b marks the 15th percentile?

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Then $\frac{y-15}{2} = z \approx 1.04, so\ y \approx 17.08.$

Recall that normal distributions are symmetric around the mean, where P(Y ≤ μ) = 0.5. Since

$$|P(Y \leq \mu) - P(Y \leq y)| = |0.5 - 0.85| = 0.35 = |P(Y \leq \mu) - P(Y \leq b)|,$$
$$b = \mu - |b - \mu| = 15 - |17.08 - 15| = 12.92,$$

so $b \approx 12.92$.

Problem 7 – Tweets

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A prolific twitter user tweets approximately 350 tweets per week. Let's assume for simplicity that the tweets are independent, and each consists of a uniformly random number of characters between 10 and 140. (Note that this is a discrete uniform distribution.) Thus, the central limit theorem (CLT) implies that the number of characters tweeted by this user is approximately normal with an appropriate mean and variance. Assuming this normal approximation is correct, estimate the probability that this user tweets between 26,000 and 27,000 characters in a particular week. (This is a case where continuity correction will make virtually no difference in the answer, but you should still use it to get into the practice!).

Work on this with the people around you and then we'll go over it together!

Problem 7 – Tweets

- Let X be the total number of characters tweeted by a twitter user in a week. Let $X_i \sim Unif(10, 140)$ be the number of characters in the ith tweet (since the start of the week).

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    - X is the sum of 350 IID RVs with $\mu = 75$ and $\sigma^2 = 1430$ (recall that $E[X] = \frac{a+b}{2}$ and $Var(X) = \frac{(b-a+1)^2-1}{12}$ for the discrete uniform RV), so $X \approx N \sim Normal(350 \cdot 75, 350 \cdot 1430)$.

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- We find: $P(26{,}000 \leq X \leq 27{,}000) = P(25{,}999.5 \leq X \leq 27{,}000.5)$
  - Notice the continuity correction is an equality, *not* an approximation

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- We find: $P(26{,}000 \leq X \leq 27{,}000) = P(25{,}999.5 \leq X \leq 27{,}000.5)$
  - Apply CLT: $P(25{,}999.5 \leq X \leq 27{,}000.5) \approx P(25{,}999.5 \leq N \leq 27{,}000.5)$
  - Standardize: $P(25{,}999.5 \leq N \leq 27{,}000.5) = P\left(\frac{25999.5-350\cdot 75}{\sqrt{350\cdot 1430}} \leq \frac{N-350\cdot 75}{\sqrt{350\cdot 1430}} \leq \frac{27000.5-350\cdot 75}{\sqrt{350\cdot 1430}}\right)$
    - Note this is an <u>equality</u>, *not* an approximation

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  - Standardize: $P(25{,}999.5 \leq N \leq 27{,}000.5) = P(\frac{25999.5-350\cdot 75}{\sqrt{350\cdot 1430}} \leq \frac{N-350\cdot 75}{\sqrt{350\cdot 1430}} \leq \frac{27000.5-350\cdot 75}{\sqrt{350\cdot 1430}})$
  - $\approx P\left(-0.3541 \leq \frac{N-350\cdot 75}{\sqrt{350\cdot 1430}} \leq 1.0608\right)$ (approximation due to rounding)
  - This is $= \Phi(1.0608) - \Phi(-0.3541) \approx 0.4923$ (equals, then approximation due to rounding in Φ table)

Problem 5 – Round off error

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- Let X be the sum of 100 real numbers, and let Y be the same sum, but with each number rounded to the nearest integer before summing. If the roundoff errors are independent and uniformly distributed between -0.5 and 0.5, what is the approximate probability that $|X - Y| > 3$?

Work on this with the people around you and then we'll go over it together!

Problem 5 – Round off error

- Notation: $X = \sum_{i=1}^{100} X_i$ and $Y = \sum_{i=1}^{100} r(X_i)$, where $r(\cdot)$ rounds to the nearest integer. Then

$$X - Y = \sum_{i=1}^{100} X_i - r(X_i)$$

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- Each $X_i - r(X_i)$ is round off error which is distributed as $Uni(-0.5, 0.5)$. We know the expectation and variance of a continuous uniform distribution: $\mu = 0$ and $\sigma^2 = \frac{1}{12}$

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$$X - Y = \sum_{i=1}^{100} X_i - r(X_i)$$
- Each $X_i - r(X_i)$ is round off error which is distributed as $Uni(-0.5, 0.5)$. We know the expectation and variance of a continuous uniform distribution: $\mu = 0$ and $\sigma^2 = \frac{1}{12}$
- We then use the CLT: $X - Y \approx W \sim N\left(0, \frac{100}{12}\right)$
- Standardize:
$$P(|X - Y| > 3) \approx P(|W| > 3) = 2P(W > 3) \approx 2P(Z > 1.039)$$
$$\approx 0.29834$$

Problem 11 –
Min and max of i.i.d random variables

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- Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables each with CDF $F_X(x)$ and pdf $f_X(x)$. Let $Y = \min(X_1, \ldots, X_n)$ and let $Z = \max(X_1, \ldots, X_n)$. Show how to write the CDF and PDF of Y and Z in terms of the functions $F_X(\cdot)$ and $f_X(\cdot)$.

Work on this with the people around you and then we'll go over it together!

Problem 11 – Min and max of i.i.d random variables

- Compute the CDFs of Z and Y (chain of equivalences):

$$F_Z(z) = P(Z < z) = P(X_1 < z, \dots, X_n < z)$$
$$= P(X_1 < z) \cdot \dots \cdot P(X_n < z) = \left(F_X(z)\right)^n$$

$$F_Y(y) = P(Y < y) = 1 - P(Y > y) = 1 - P(X_1 > y, \dots, X_n > y)$$
$$= 1 - P(X_1 > y) \cdot \dots \cdot P(X_n > y) = 1 - \left(1 - F_X(y)\right)^n$$

Note that the second to last step follows due to IID-ness of X_i's.

Problem 11 – Min and max of i.i.d random variables

- Compute the CDFs of Z and Y (chain of equivalences):

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$$= P(X_1 < z) \cdot \dots \cdot P(X_n < z) = \left(F_X(z)\right)^n$$

$$F_Y(y) = P(Y < y) = 1 - P(Y > y) = 1 - P(X_1 > y, \dots, X_n > y)$$
$$= 1 - P(X_1 > y) \cdot \dots \cdot P(X_n > y) = 1 - \left(1 - F_X(y)\right)^n$$

Note that the second to last step follows due to IID-ness of X_i's.

- Now compute the PDFs of Z and Y:

$$f_Z(z) = \frac{d}{dz} F_Z(z) = \frac{d}{dz}\left(F_X(z)\right)^n$$

$$= n \cdot F_X(z)^{n-1} \cdot \frac{d}{dz} F_X(z) = n \cdot F_X(z)^{n-1} \cdot f_X(z)$$

By similar logic, $f_Y(y) = \frac{d}{dy} F_Y(y) = n \cdot \left(1 - F_X(y)\right)^{n-1} \cdot f_X(y)$

That's All, Folks!

Thanks for coming to section this week!
Any questions?