CSE 312 Section 3

Conditional Probability

Administrivia

Announcements & Reminders

- HW1
  - Grades released on gradescope – check your submission to read comments
  - Regrade requests open ~24 hours after grades are released and close after a week

- HW2
  - Was due yesterday, Wednesday 1/22 @ 11:59pm
  - Late deadline Saturday 1/25 @ 11:59pm (max of 3 late days per problem)

- HW3
  - Released on the course website
  - Due Wednesday 1/29 @ 11:59pm

Review

Any lingering questions from this last week?

Each week in section, we’ll be reviewing the main concepts from this week and putting them into action by going through some practice problems together. But before we get into that review, we’ll try to start off each section with some time for you to ask questions. Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

Review Questions?

Question #1

It is always the case that $P(A|B) = P(B|A)$

- True
- False

Question #1

It is always the case that $P(A|B) = P(B|A)$

- True
- False

Question #2

Suppose A and B are independent events. Then:

- $P(A \cap B) = P(A)P(B)$
- $P(A|B) = P(A)$
- Both are true
- Both are false

Question #2

Suppose A and B are independent events. Then:

- $P(A \cap B) = P(A)P(B)$
- $P(A|B) = P(A)$
- Both are true
- Both are false

Question #3

For any two events A and B:

- $P(A|B) = P(A \cap B)$
- $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$
- $P(A|B) = \dfrac{P(B|A)P(B)}{P(A)}$

Question #3

For any two events A and B:

- $P(A|B) = P(A \cap B)$
- $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- $P(A|B) = \frac{P(B|A)P(B)}{P(A)}$

Question #4

Let A and B be the event that a six-sided die is at most 3 and at least 4, respectively. Then A and B are a partition.

- True
- False

Question #4

Let A and B be the event that a six-sided die is at most 3 and at least 4, respectively. Then A and B are a partition.

- True
- False

Question #5

Let T be the event that Alice tests positive for the cold. Let C be the event that Alice actually has the cold. Suppose the probability that Alice tests positive given that she has a cold is 0.8. Then the probability she tests negative given that she has a cold is:

- 0.8
- 0.2
- Not enough information

Question #5

Let T be the event that Alice tests positive for the cold. Let C be the event that Alice actually has the cold. Suppose the probability that Alice tests positive given that she has a cold is 0.8. Then the probability she tests negative given that she has a cold is:

- 0.8
- 0.2
- Not enough information

Question #6

For events $A_1, \ldots, A_n$ it follows that

$$P(A_1 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_2 \cap A_1) \cdot \ldots \cdot P(A_n|A_1 \cap \cdots \cap A_{n-1})$$

- True
- False

Question #6

For events $A_1, \dots, A_n$ it follows that

$$P(A_1 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_2 \cap A_1) \cdot \dots \cdot P(A_n|A_1 \cap \cdots \cap A_{n-1})$$

- True
- False

Problem 4 – Game Show

4 – Game Show

Corrupted by their power, the judges running the popular game show America's Next Top Mathematician have been taking bribes from many of the contestants. During each of two episodes, a given contestant is either allowed to stay on the show or is kicked off. If the contestant has been bribing the judges, she will be allowed to stay with probability 1. If the contestant has not been bribing the judges, she will be allowed to stay with probability 1/3, independent of what happens in earlier episodes. Suppose that 1/4 of the contestants have been bribing the judges. The same contestants bribe the judges in both rounds.

4 – Game Show

2 rounds, contestants bribe in both rounds. If contestant has been bribing , allowed to stay with probability 1. If contestant has not been bribing, allowed to stay with probability 1/3, independent of earlier. Suppose that 1/4 of the contestants have been bribing the judges.

a) If you pick a random contestant, what is the probability that she is allowed to stay during the first episode?
b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?
c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?
d) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

Work on this problem with the people around you, and then we'll go over it together!

4 – Game Show

a) If you pick a random contestant, what is the probability that she is allowed to stay during the first episode?

4 – Game Show

a) If you pick a random contestant, what is the probability that she is allowed to stay during the first episode?

Let S_i be the event that she stayed during the i-th episode. By the Law of Total Probability conditioning on whether the contestant bribed the judges we get,

$$\mathbb{P}(S_1) = \mathbb{P}(\text{Bribe})\mathbb{P}(S_1|\text{Bribe}) + \mathbb{P}(\text{NoBribe})\mathbb{P}(S_1|\text{NoBribe})$$

4 – Game Show

a) If you pick a random contestant, what is the probability that she is allowed to stay during the first episode?

Let S_i be the event that she stayed during the i-th episode and B be the event that she bribes the judges. By the Law of Total Probability conditioning on whether the contestant bribed the judges we get,

$$\begin{aligned}
&\mathbb{P}(S_1) = \mathbb{P}(\mathrm{B})\mathbb{P}(S_1|\mathrm{Bribe}) + \mathbb{P}(B^C)\mathbb{P}(S_1|\mathrm{NoBribe}) \\
&= \frac{1}{4}\cdot 1 + \frac{3}{4}\cdot\frac{1}{3} \\
&= 1/2
\end{aligned}$$

4 – Game Show

b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?

4 – Game Show

b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?

The event is $S_1 \cap S_2$. By the Law of Total Probability we get,

$$\mathbb{P}(S_1 \cap S_2) = \mathbb{P}(\text{Bribe})\mathbb{P}(S_1 \cap S_2|\text{Bribe}) + \mathbb{P}(\text{NoBribe})\mathbb{P}(S_1 \cap S_2|\text{NoBribe})$$

4 – Game Show

b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?

The event is $S_1 \cap S_2$. By the Law of Total Probability we get,

$$\mathbb{P}(S_1 \cap S_2) = \mathbb{P}(\mathrm{B})\mathbb{P}(S_1 \cap S_2|\mathrm{B}) + \mathbb{P}(\mathrm{B}^\mathrm{C})\mathbb{P}(S_1 \cap S_2|\mathrm{B}^\mathrm{C})$$

If she hasn't been bribing judges, then the probability she stays on the show is 1/3, independent of what happens on earlier episodes. By conditional independence, we have:

$$= \frac{1}{4} \cdot 1 + \mathbb{P}(\mathrm{B}^\mathrm{C})\mathbb{P}(S_1|\mathrm{B}^\mathrm{C})\mathbb{P}(S_2|\mathrm{B}^\mathrm{C})$$

4 – Game Show

b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?

The event is $S_1 \cap S_2$. By the Law of Total Probability we get,

$$\mathbb{P}(S_1 \cap S_2) = \mathbb{P}(\mathrm{B})\mathbb{P}(S_1 \cap S_2|\mathrm{B}) + \mathbb{P}(\mathrm{B}^{\mathrm{C}})\mathbb{P}(S_1 \cap S_2|\mathrm{B}^{C})$$

If she hasn't been bribing judges, then the probability she stays on the show is 1/3, independent of what happens on earlier episodes. By conditional independence, we have:

$$= \frac{1}{4} \cdot 1 + \mathbb{P}(\mathrm{B}^{\mathrm{C}})\mathbb{P}(S_1|\mathrm{B}^{\mathrm{C}})\mathbb{P}(S_2|\mathrm{B}^{\mathrm{C}})$$
$$= \frac{1}{4} \cdot 1 + \frac{3}{4} \cdot \frac{1}{3} \cdot \frac{1}{3}$$
$$= \frac{1}{3}$$

4 – Game Show

c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

4 – Game Show

c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

By the definition of conditional probability and the Law of Total Probability,

$$\mathbb{P}(\overline{S_2}|S_1) = \frac{\mathbb{P}(S_1 \cap \overline{S_2})}{\mathbb{P}(S_1)} = \frac{\mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B})\mathbb{P}(\mathrm{B}) + \mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B}^{\mathrm{C}})\mathbb{P}(\mathrm{B}^{\mathrm{C}})}{\mathbb{P}(S_1)}$$

4 – Game Show

c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

By the definition of conditional probability and the Law of Total Probability,

$$\mathbb{P}(\overline{S_2}|S_1) = \frac{\mathbb{P}(S_1 \cap \overline{S_2})}{\mathbb{P}(S_1)} = \frac{\mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B})\mathbb{P}(\mathrm{B}) + \mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B}^C)\mathbb{P}(\mathrm{B}^C)}{\mathbb{P}(S_1)}$$

If she has been bribing judges, then she is guaranteed to stay on the show, so

$$\mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B}) = \mathbb{P}(S_1|\mathrm{B})\mathbb{P}(\overline{S_2}|\mathrm{B}) = 1 \cdot 0 = 0$$

4 – Game Show

c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

By the definition of conditional probability and the Law of Total Probability,

$$\mathbb{P}(\overline{S_2}|S_1) = \frac{\mathbb{P}(S_1 \cap \overline{S_2})}{\mathbb{P}(S_1)} = \frac{\mathbb{P}(S_1 \cap \overline{S_2}|\text{Bribe})\mathbb{P}(\text{Bribe}) + \mathbb{P}(S_1 \cap \overline{S_2}|\text{NoBribe})\mathbb{P}(\text{NoBribe})}{\mathbb{P}(S_1)}$$

If she has been bribing judges, then she is guaranteed to stay on the show, so

$$\mathbb{P}(S_1 \cap \overline{S_2}|\text{B}) = \mathbb{P}(S_1|\text{B})\mathbb{P}(\overline{S_2}|\text{B}) = 1 \cdot 0 = 0$$

If she hasn't been bribing, then the probability she leaves is 2/3 by complementing, so

$$\mathbb{P}(S_1 \cap \overline{S_2}|\text{B}^\text{C}) = \mathbb{P}\left(S_1|\text{B}^\text{C}\right)\mathbb{P}(\overline{S_2}|\text{B}^C) = \frac{1}{3}\cdot\frac{2}{3}$$

4 – Game Show

c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

We already computed $\mathbb{P}(S_1)$ in part (a). Putting it all together we get:

$$\mathbb{P}(\overline{S_2}|S_1) = \frac{\mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B})\mathbb{P}(\mathrm{B}) + \mathbb{P}(S_1 \cap \overline{S_2}|\mathrm{B}^C)\mathbb{P}(\mathrm{B}^C)}{\mathbb{P}(S_1)}$$

$$= \frac{0 \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{3}{4}}{\frac{1}{2}}$$

$$= \frac{1/6}{1/2}$$

$$= \frac{1}{3}$$

4 – Game Show

d) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

4 – Game Show

d) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

Let B be the event that she bribed the judges. By Bayes' Theorem,

$$\mathbb{P}(B|S_1) = \frac{\mathbb{P}(S_1|B)\mathbb{P}(B)}{\mathbb{P}(S_1)} = \frac{1 \cdot \frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Bayes Theorem with Law of Total Probability

- $$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\sum_i P(B|E_i)P(E_i)}$$

Where $E_1, \dots, E_n$ are a set of mutually exclusive events that partition the sample space (i.e. $\sum_i P(E_i) = 1$). We use the law of total probability to find $P(B)$.

Bayes Theorem with Law of Total Probability

- $$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\sum_i P(B|E_i)P(E_i)}$$

Where $E_1, \ldots, E_n$ are a set of mutually exclusive events that partition the sample space (i.e. $\sum_i P(E_i) = 1$). We use the law of total probability to find $P(B)$. A common structure you'll come across is:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^C)P(A^C)}$$

Problem 5 – Parallel Systems

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

b) If the system is functioning, what is the probability that component 1 is working?

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

Work on this problem with the people around you, and then we'll go over it together!

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

Let C_i be the event component i is working, and F be the event that the system is functioning. For the system to function, it is sufficient for any component to be working. This means that the only case in which the system does not function is when none of the components work. We can then use complementing to compute $\mathbb{P}(F)$, knowing that $\mathbb{P}(C_i) = p$. We get:

$$\mathbb{P}(F) = 1 - \mathbb{P}(F^C) = 1 - \mathbb{P}\left(\bigcap_{i=1}^{n} C_i^C\right) = 1 - \prod_{i=1}^{n} \mathbb{P}(C_i^C)$$

$$= 1 - \prod_{i=1}^{n} \left(1 - \mathbb{P}(C_i)\right) = 1 - \prod_{i=1}^{n} (1 - p) = 1 - (1 - p)^n$$

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

b) If the system is functioning, what is the probability that component 1 is working?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

b) If the system is functioning, what is the probability that component 1 is working?

We know that for the system to function only one component needs to be working, so for all i, we have $\mathbb{P}(F|C_i) = 1$. Using Bayes Theorem, we get

$$\mathbb{P}(C_1|F) = \frac{\mathbb{P}(F|C_1)\mathbb{P}(C_1)}{\mathbb{P}(F)} = \frac{1 \cdot p}{1-(1-p)^n} = \frac{p}{1-(1-p)^n}$$

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

$$\mathbb{P}(C_1|C_2, F) = \mathbb{P}(C_1|C_2) = \mathbb{P}(C_1) = p$$

The first equality holds because knowing C_2 and F is just as good as knowing C_2 (since if C_2 happens, F does too), and the second equality holds because the components working are independent of each other.

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

More formally, we can use the definition of conditional probability along with a careful application of the chain rule to get the same result. We start with the following expression:

$$\mathbb{P}(C_1|C_2,F) = \frac{\mathbb{P}(C_1,C_2,F)}{\mathbb{P}(C_2,F)} = \frac{\mathbb{P}(F|C_1,C_2)\mathbb{P}(C_1|C_2)\mathbb{P}(C_2)}{\mathbb{P}(F|C_2)\mathbb{P}(C_2)}$$

We note that the system is guaranteed to work if any one component is working, so $\mathbb{P}(F|C_1,C_2) = \mathbb{P}(F|C_2) = 1$. We also note that components work independently of each other, hence $\mathbb{P}(C_1|C_2) = \mathbb{P}(C_1)$. With that in mind, we can rewrite our expression so that:

$$\mathbb{P}(C_1|C_2,F) = \frac{1 \cdot \mathbb{P}(C_1)\mathbb{P}(C_2)}{1 \cdot \mathbb{P}(C_2)} = \mathbb{P}(C_1) = p$$

Problem 9 – Dependent Dice Duo

9 – Dependent Dice Duo

This problem demonstrates that independence can be “broken” by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event “$D_1 = 1$”, F be the event “$D_2 = 6$”, and G be the event “$D_1 + D_2 = 7$”. Even though E and F are independent, show that

$$\mathbb{P}(E \cap F|G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$$

Work on this problem with the people around you, and then we’ll go over it together!

9 – Dependent Dice Duo

This problem demonstrates that independence can be “broken” by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event “$D_1 = 1$”, F be the event “$D_2 = 6$”, and G be the event “$D_1 + D_2 = 7$”. Even though E and F are independent, show that $\mathbb{P}(E \cap F|G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that $\mathbb{P}(E \cap F | G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

$$\mathbb{P}(E|G) = \mathbb{P}(D_1 = 1 | D_1 + D_2 = 7) = \frac{1}{6}$$

$$\mathbb{P}(F|G) = \mathbb{P}(D_2 = 6 | D_1 + D_2 = 7) = \frac{1}{6}$$

$$\mathbb{P}(E \cap F|G) = \mathbb{P}(D_1 = 1 \cap D_2 = 6 | D_1 + D_2 = 7) = \frac{1}{6}$$

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning. Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that $\mathbb{P}(E \cap F | G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$

When we condition on G, our sample space Ω becomes {(1,6), (2,5), (3,4), (4,3), (2,5), (6,1)}, where the first number in the pair is the D_1 outcome and the second number is the D_2 outcome.

From here we can see that $P(E|G) = P(D_1 = 1 \mid D_1 + D_2 = 7) = 1/6$ as (1,6) is 1 of the 6 possible rolls that sum to 7 and each roll is equally likely.

We also see that $P(F|G) = P(D_2 = 6 \mid D_1 + D_2 = 7) = 1/6$ and $P(E \cap F|G) = P(D_1 = 1 \cap D_2 = 6 \mid D_1 + D_2 = 7) = 1/6$ using similar reasoning. Now we have that $P(E|G) * P(F|G) = 1/36$. Notice that $1/36 \neq 1/6$ so we have shown that independence can be "broken" by conditioning.

Problem 6 – Allergy Season

6 – Allergy Season

In a certain population, everyone is equally susceptible to colds. The number of colds suffered by each person during each winter season ranges from 0 to 4, with probability 0.2 for each value (see table below). A new cold prevention drug is introduced that, for people for whom the drug is effective, changes the probabilities as shown in the table. Unfortunately, the effects of the drug last only the duration of one winter season, and the drug is only effective in 20% of people, independently.

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

6 – Allergy Season

The drug is only effective in 20% of people, independently.

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

a) Sneezy decides to take the drug. Given that he gets 1 cold that winter, what is the probability that the drug is effective for Sneezy?

b) The next year he takes the drug again. Given that he gets 2 colds in this winter, what is the updated probability that the drug is effective for Sneezy?

c) Why is the answer to (b) the same as the answer to (a)?

Work on this problem with the people around you, and then we'll go over it together!

6 – Allergy Season

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

a) Sneezy decides to take the drug. Given that he gets 1 cold that winter, what is the probability that the drug is effective for Sneezy?

6 – Allergy Season

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

a) Sneezy decides to take the drug. Given that he gets 1 cold that winter, what is the probability that the drug is effective for Sneezy?

Let E be the event that the drug is effective for Sneezy, and C_i be the event that he gets i colds the first winter. By Bayes' Theorem,

$$\mathbb{P}(E|C_1) = \frac{\mathbb{P}(C_1|E)\mathbb{P}(E)}{\mathbb{P}(C_1|E)\mathbb{P}(E) + \mathbb{P}(C_1|\bar{E})\mathbb{P}(\bar{E})} = \frac{0.3 \cdot 0.2}{0.3 \cdot 0.2 + 0.2 \cdot 0.8} = \frac{3}{11}$$

6 – Allergy Season

b) The next year he takes the drug again. Given that he gets 2 colds in this winter, what is the updated probability that the drug is effective for Sneezy?

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

6 – Allergy Season

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

b) The next year he takes the drug again. Given that he gets 2 colds in this winter, what is the updated probability that the drug is effective for Sneezy?

Let the reduced sample space for part (b) be C_1 from part (a), so that $\mathbb{P}_{C_1}(E) = \mathbb{P}_\Omega(E|C_1)$. Let D_i be the event that he gets i colds the second winter. By Bayes' Theorem,

$$\mathbb{P}(E|D_2) = \frac{\mathbb{P}(D_2|E)\mathbb{P}(E)}{\mathbb{P}(D_2|E)\mathbb{P}(E) + \mathbb{P}(D_2|\bar{E})\mathbb{P}(\bar{E})} = \frac{0.2 \cdot \frac{3}{11}}{0.2 \cdot \frac{3}{11} + 0.2 \cdot \frac{8}{11}} = \frac{3}{11}$$

6 – Allergy Season

c) Why is the answer to (b) the same as the answer to (a)?

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

6 – Allergy Season

c) Why is the answer to (b) the same as the answer to (a)?

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

The probability of two colds whether or not the drug was effective is the same. Hence knowing that Sneezy got two colds does not change the probability of the drug's effectiveness.

Problem 3 – Marbles in Pockets

3 – Marbles in Pockets

Aleks has 5 blue and 3 white marbles in her left pocket, and 4 blue and 4 white marbles in her right pocket. If he transfers a randomly chosen marble from left pocket to right pocket without looking, and then draws a randomly chosen marble from her right pocket, what is the probability that it is blue?

Work on this problem with the people around you, and then we'll go over it together!

3 – Marbles in Pockets

Aleks has 5 blue and 3 white marbles in her left pocket, and 4 blue and 4 white marbles in her right pocket. If he transfers a randomly chosen marble from left pocket to right pocket without looking, and then draws a randomly chosen marble from her right pocket, what is the probability that it is blue?

3 – Marbles in Pockets

Aleks has 5 blue and 3 white marbles in her left pocket, and 4 blue and 4 white marbles in her right pocket. If he transfers a randomly chosen marble from left pocket to right pocket without looking, and then draws a randomly chosen marble from her right pocket, what is the probability that it is blue?

Let $W\_, B\_$ denote the event that we choose a white marble or a blue marble respectively, with subscripts L, R indicating from which pocket we are picking – left and right, respectively. We know that we will pick from the left pocket first, and right pocket second. We can then use the Law of Total Probability conditioning on the color of the transferred marble so that:

$$\mathbb{P}(B_R) = \mathbb{P}(W_L) \cdot \mathbb{P}(B_R|W_L) \cdot \mathbb{P}(B_L) \cdot \mathbb{P}(B_R|B_L) = \frac{3}{8} \cdot \frac{4}{9} + \frac{5}{8} \cdot \frac{5}{9} = \frac{37}{72}$$

Problem 7 – A Game

7 – A Game

Howard and Jerome are playing the following game: A 6-sided die is thrown and each time it's thrown, regardless of the history, it is equally likely to show any of the six numbers.

- If it shows 5, Howard wins.
- If it shows 1, 2, or 6, Jerome wins.
- Otherwise, they play a second round and so on.

What is the probability that Jerome wins on the 4th round?

Work on this problem with the people around you, and then we'll go over it together!

7 – A Game

Howard and Jerome are playing the following game: A 6-sided die is thrown and each time it's thrown, regardless of the history, it is equally likely to show any of the six numbers.
If it shows 5, Howard wins. If it shows 1, 2, or 6, Jerome wins. Otherwise, they play a second round and so on. What is the probability that Jerome wins on the 4th round?

Let S_i be the event that Jerome wins on the i-th round and let N_i be the event that nobody wins on the i-th round. Then we are interested in the event

$$N_1 \cap N_2 \cap N_3 \cap S_4$$

Using the chain rule, we have

$$\mathbb{P}(N_1 \cap N_2 \cap N_3 \cap S_4) = \mathbb{P}(N_1) \cdot \mathbb{P}(N_2|N_1) \cdot \mathbb{P}(N_3|N_1, N_2) \cdot \mathbb{P}(S_4|N_1, N_2, N_3) = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}$$

In the final step, we used the fact that if the game hasn't ended, then the probability that it continues for another round is the probability that the die comes up 3 or 4, which has probability 1/3.

That's All, Folks!

Thanks for coming to section this week!
Any questions?